PRELIMINARY DRAFT

Cheers to Good Health of the U.S. Short-Run Phillips Curve: Output and Inflation from 1960 to 2012

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Abstract

In this paper a state of the U.S. short-run Phillips curve from 1960 to 2012 is examined. Phillips curve, or Phillips correlation, seems very much alive and of good health. I demonstrate that a co-movement between output, unemployment, capacity utilization and measures of core inflation is both strong and stable if viewed at periodicity consistent with inflation cycles, i.e. deviation of inflation from its long term expectations. One principal component (factor) drives the cycle in U.S. real and nominal data. To account for endogeniety of output and inflation, a simple forward-looking economic model is build and used to carry out counterfactual simulations and to provide a historical interpretation of the data. The stable co-movement of output with inflation also provides further evidence against the standard version of the real business cycles hypothesis. There are policy implications of estimated inflation-relevant output cycle, since it implies that the inflation-relevant output gap is almost closed.

Keywords: Phillips curve, NAIRU, output gap DSGE model, state-space
JEL Codes: C10, E50

1. Introduction

In this paper I pursue the nature of relationship between 'inflation' and cyclical economic activity, relationship often labelled as *short-run Phillips correlation*, or a Phillips curve. The paper demonstrates that short-run US Phillips curve is of good health and one can reiterate J. Fuhrer's claim that "the Phillips curve is alive and well", see Fuhrer (1995). The strength of empirical regularities across business cycles is striking and it recent events during Great Recession seem to vibe relatively well with the theory of the forward-looking Phillips curve and New Keynesian Phillips Curve.

Recently, also in relationship with the Great Recession, there have been some sceptical views of the link between output, unemployment and inflation and forecasting performance based on Phillips correlation. For instance, Ball and Mazumder (2011) argue that Great Recession provides the evidence against the New Keynesian Phillips Curve with rational expectations. Stock and Watson (2008) present suite of models having difficulties to forecast inflation, arguing that univariate or simple statistical models dominate the more structural ones based on economic theory of the Phillips curve.

I follow a very simple principle of indirect measurement. Instead of proposing a measure of output gap and linking it to inflation, I use the inflation as the key to determination of the output gap and NAIRU (Non-Accelerating Inflation Rate of Unemployment), not vice versa. To do so, a stability of the relationship needs to be established. Deviation of inflation from its long-term expectations is used to find the periodicity at which output co-moves closely and in a stable way with inflation. The co-movement is found to be strong and relatively stable. An alternative approach

[☆] The views expressed herein are those of the author and should not be attributed to the International Monetary Fund, its Executive Board, or its management.

[1] I would like to thank to Jan Brůha for comments.

November 9, 2012

would be to assume a particular measure of cycle, most likely estimated without reference to inflation, and use it as an explanatory source for inflation, perhaps with considerations for nonlinear, asymmetric and time-varying relationship.

The *economic theory* view of the paper is based on modern understanding of relationship of deviation from unemployment (and output) from its 'natural' or structural level with deviation of inflation from its equilibrium – that is explicit or implicit inflation target. Empirical analysis of the paper suggest there is a strong relationship between deviation from its long-term expectations and output cycle. Using a simple forward-looking model I evaluate a hypothesis that bulk of the cyclical variation in US core inflation is due to demand shocks and carry out counter-factual simulations using the model. The notion of 'target' or 'trend' inflation is crucial for understanding the short-run Phillips curve, viewing the long-run Phillips curve as vertical. I discuss the notion of implicit inflation target for the US economy, making use of long-term inflation expectation survey, long-term yields and evidence from Federal Open Market Committee (FOMC) transcripts.

The *time series analysis* point of view of the paper rests on an agnostic spectral analysis of macroeconomic time series, exploring coherence among time series across frequencies. I find strong coherence among inflation and economic activity at business cycle frequencies, often defined as cycles with periodicity between 6–32 quarters, or 8–32 quarters. Using the data for long-term inflation expectations I search for length of the cycle that corresponds to deviation of core inflation from the implicit target and evaluate co-movement of this variable with output gap at that periodicity. Up to a periodicity o 15 years the 'Phillips correlation' is surprisingly strong among inflation, output, unemployment and capacity utilisation.

Both views, theory and time series, are mutually consistent, complementary and highlight the importance of considering the long term inflation expectations as a factor to judge inflation with respect to. As one would assume that inflation target does not change abruptly, the deviation of inflation from the target produces cyclical measure of inflation, which can be explained by changes in real economic activity with success.[2] On the other end of the spectrum (literally) the inflation features non-persistent, hard-to explain high frequency variations coming from multiple sources, not all explainable by fundamentals. There are many concepts of core inflation, though all have in common a reduction in high-frequency variance of inflation.

The inflation is perceived as composed of three distinct, well identified components. The first is a *trend component* coincident with long-term inflation expectations and thus a perceived inflation target. The second is a *cyclical component* reflecting the cyclical disturbances and the reaction of the economy. The third component boils down to either explain or abstract from the *high-frequency non-persistent variations*, which are either first-order impact of oil prices, changes in relative prices and mismeasurements. This component, present even in a core inflation, is not required to be explained by fundamental cyclical factors in this study.

The following are the results of the paper. First, the 'Phillips correlation' is quite robust and stable feature of US economy at cyclical frequencies, broadly defined. Second, using the inflation cycle as a guidance to estimate the output gap (or NAIRU), it is clear that inflation-relevant output is smaller than most estimates of the output gap by policy institutions and academia and is closing up in the U.S. Third, a parsimonious forward-looking model dynamics is consistent with empirical facts on US shot-run Phillips curve when path of long-run implicit inflation target is accounted for.

The approach of the paper has close connection to other contributions to the literature. It has been recognized that trend inflation or implicit inflation target is an important part of the story in the United States, see e.g. Ireland (2007), or Cogley, Primiceri, and Sargent (2010). The identification of the Phillips curve and the strength of correlation at business cycle and long frequencies was analyzed in King and Watson (1994), who use vector-autoregressive models to analyze the data.

2. Data Analysis – Time & Frequency domain

2.1. Data Choice and Methods

The choice of the data and data transformations are guided by economic intuition and theory. The data set for empirical analysis consists of real GDP, capacity utilization, unemployment rate and multiple inflation measures. The

[2]The concept of inflation 'gap' is not completely appealing at the beginning of cold-turkey disinflations, where a positive inflation gap would correspond to negative cycle in output and unemployment. Surprisingly, even under the Volcker disinflation the co-movement of both 'gaps' is reasonable. This is due to the fact that long-term inflation expectations of private households and businesses only gradually followed an implicit, not communicated true target of the FED.

headline inflation is BLS CPI-U series and as measures for core inflation I consider the following candidates: (i) headline inflation less food and energy, (ii) FRB Dallas trimmed-mean inflation, (iii) FRB Cleveland 16% trimmed mean (revised) series and (iv) FRB Cleveland median inflation. All growth rates and inflations are quarter over quarter (q/q), annualized – unless explicitly stated otherwise. The FRB Cleveland revised measures of core inflation are not available before 1983, but an original core inflation measures at FRB Cleveland can be used to extend the series to February 1967.

Searching for a short-run Phillips curve implies a notion of frequency – *short-run.* Thus, I view the data through a common lenses – business cycle frequencies ranging from 0–32 or 8-32 quarters durations. Eventually the frequencies up to 60 quarters are considered to match output cycles with inflation cycles. I do not put necessarily the meaning of an equilibrium or a natural level to these trend and gaps estimates, as ex-ante it is not clear that natural rate of unemployment should move together with natural output at same frequencies. Further, it is naive to assume that economic forces affecting structural/equilibrium unemployment affect strictly a sub-set of frequencies.

To approximate the ideal band-pass filter dissecting the frequencies of interest, I use time- and frequency-domain methods – band-pass filter by Christiano and Fitzgerald (1999), high-pass Leser/Hodrick-Prescott filter, see Leser (1961) and **?** and frequency-domain filter of Iacobucci and Noullez (2005)[3]. The data are thus processed by a filtering operation $\hat{X}_t = A(L)X_t$. Obviously, a first-difference or fourth-period difference are also filters. A first-difference filters is invertible and amplifies high frequencies.

2.2. *Analysis of co-movements across cyclical frequencies*

2.2.1. *Real variables*

The relationships of GDP log-level, unemployment and capacity utilization across frequencies are depicted at Fig. 1, where *coherence* – essentially a correlation coefficient at particular frequency bands– is calculated.[4] Similartly to correlation coefficient, coherence is invariant under linear transformation of involved variables. Further, co-herence handles all phase-shifts and measures only similarity in cycles, hence coherence between x_t, y_t is equivalent to coherence between x_t, y_{t-1}.

The link between output and unemployment during business cycles is strong and very well understood, often labaled as *Okun's law* – see e.g. Okun (1962). The law is often stated in a form $Y_t - Y_t^\star = \alpha(U_t - U_t^\star)$, where $X^\star$ denotes an 'equlibrium value' of X. The linear relationship between output and unemployment is a very strong one up to frequency of 9 years, after which it starts to deteriorate, gradually. Inspecting band-pass filtered data, the period responsible is 1989–2001 for the most part.

Frequency-based Okun's law relationship suggest very close correlation at cycles up to 36 quarters, with the cyclical component of unemployment lagging 1 quarter and 1/2 of the variance of the output cyclical component. At these frequencies, Okun's law is very stron and stable from 1958-2011. Hence, the frequency-based Okun's law would be described as

$$U_t - U_t^\star = -\frac{1}{2} \times (Y_{t-1} - Y_{t-1}^\star) + \varepsilon_t. \tag{1}$$

This has important implications for modelling the relationship between output and inflation and link from NAIRU to output gap. The data clearly do not support a dynamic relationship $\hat{U}_t = \rho\hat{U}_{t-1} + (1-\rho)\alpha\hat{Y}_t + \varepsilon$, or a static relationship, which are both used in the literature, e.g. **?**.

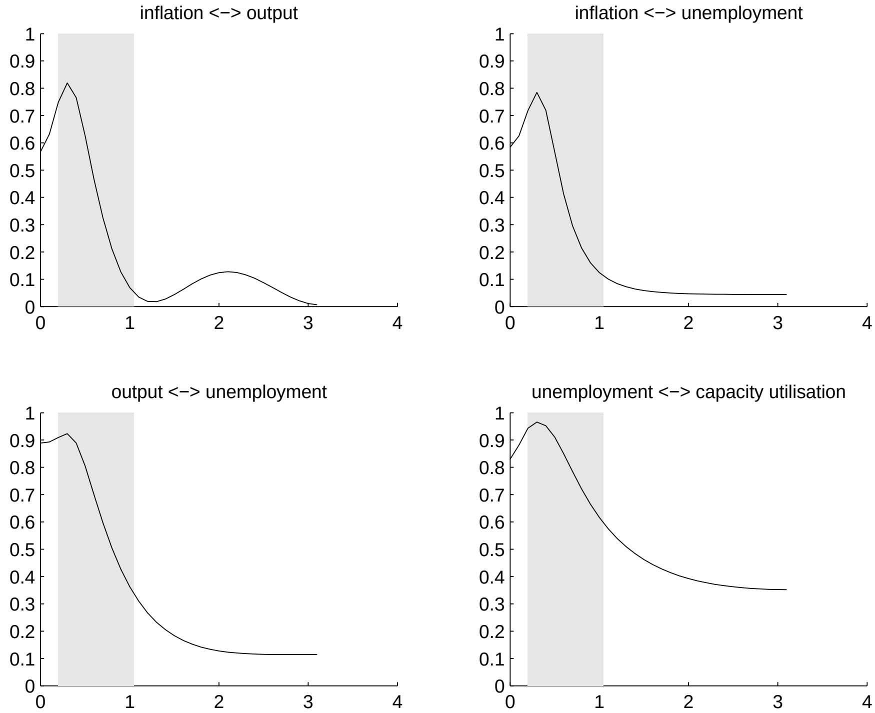
Fig. 2 depicts band-pass filtered GDP and unemployment for 1960–2011 with frequency cutoff of 8 years and 15 years, respectively. The first panel depicts the cyclical components, the other panels depict these components with unemployment shifted by one quarter and multiplied by "Okun's" coefficient of 2. The relationship between output and unemployment is one of the most robust relationship in the US data. My analysis does not fully square with statement of Gordon (2011), who argues that "procyclical productivity response of Okun's Law is obsolete in U. S. data since the mid-1980s".

Investigating co-movements of output and unemployment at different lengths of cycles is of great importance for thinking about NAIRU. The question for the theory is if there is a reason to assume cycles in unemployment being

[3]Note that Iacobucci and Noullez (2005) provide very rudimentary illustration of the filter using US Phillips curve. The frequency-domain filtering consists essentially of converting time series into its spectral representation and cutting exactly frequencies of interest and converging the result back to time domain. I use Hanning window. The frequency-domain filter by Corbae, Ouliaris, and Phillips (2002) gives similar results

[4]See appendix for coherence calculated for different sample periods.

Figure 1: Coherence – full sample



more persistent than cycles in real economic activity, e.g. what one contributes to the cycle and what is labeled as NAIRU. Data suggest that up to cycles of 15 years the co-movement between GDP and unemployment is very strong, with important exception of 1990s. Recovery after 1991 slowdown is understood as job-less recovery and so is the recovery after burst of 'dot-com' bubble after 2001. Interestingly, the co-movement of unemployment and GDP cycles up to 15 years do not suggest any major disruption in 2001 recovery.

From Fig. 1 it seems clear that there is strong linear relationship of capacity utilization and unemployment accross all frequencies, with the exception of very long frequencies – 15 years and more. At longer frequencies the downward trend in the capacity utilisation data, a survey based measure, becomes important and the relationship is corrupted. Cyclical component of capacity utilisation is twice more volatile than output cyclical components and the relationship is contemporaneous.

2.2.2. Output and inflation

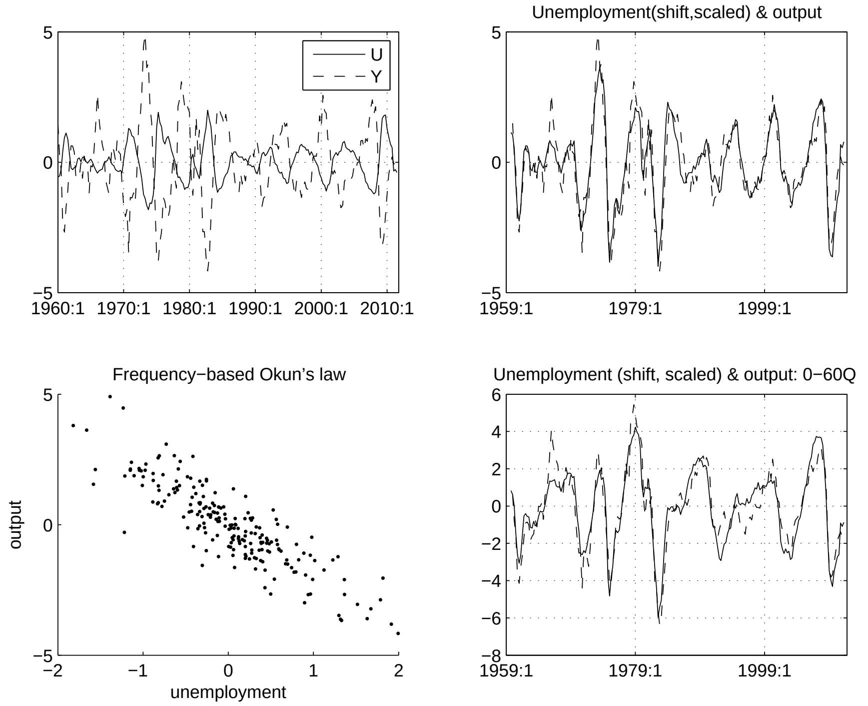
The present view of inflation is closely related to a modern view of monetary policy, with inflation targeting regimes often providing an explicit long-run nominal anchor for the economy. Consistent with vertical long-run Phillips curve, only deviation of inflation from its 'target' should be related to cyclical fluctuations in the economy.

In case of the US the central bank is endowed wit a dual mandate of monetary policy (price stability and maximum unemployment) and no *explicit inflation target*. In the data part of the paper I relate the target (trend) level of inflation to long-term inflation expectations and long frequencies of inflation with periodicity 60 quarters or more, imposing a prior that sensible central bankers would not argue for volatile changes in the target. Later, using a structural forward-looking model I estimate the implicit model-consistent core inflation target.

The longest measure of long-term inflation expectations available is the one used in FRB/US macroeconometric model of the Federal Reserve Board.[5] Since 1990Q2 the series is related to 10 years ahead inflation expectations from

[5] I would like to thank Bob Tetlow and Peter C. Chen for kindly providing me with the series. The series is identical to Survey of Professional Forecasters 10Y ahead inflation expectations since 1990, 1981–1989 is derived from Hoey survey of financial market participants, the series 1960–1980 is derived from model of bond model by Kozicki and Tinsley (2001) and thus are the least reliable part of the series.

Figure 2: Unemployment & GDP – frequency based Okun's law



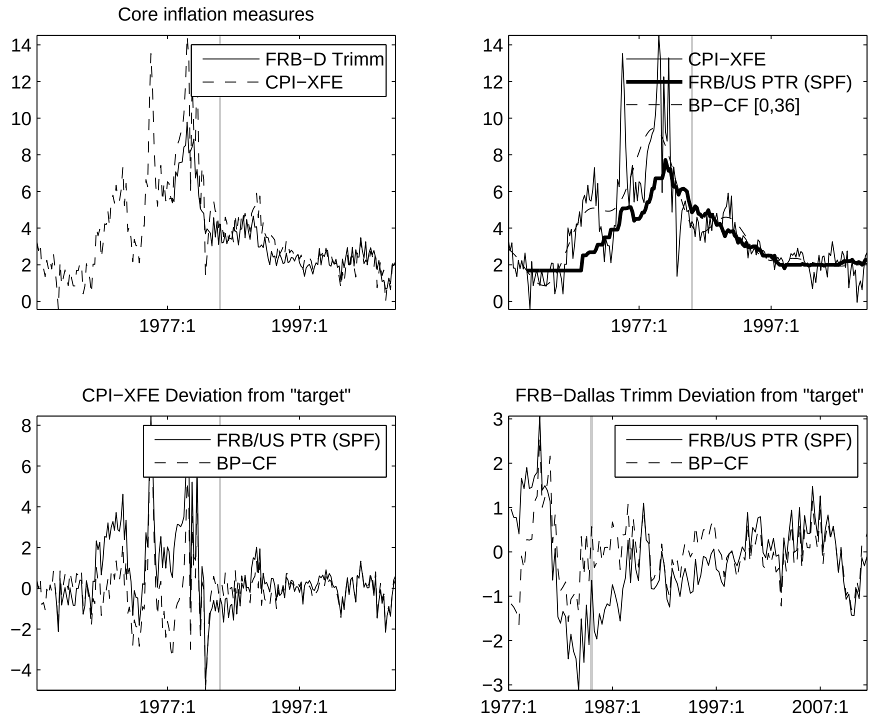
Survey of Professional Forecasters (SPF, FRB Philladelphia) and starting from 2007Q1 it coincides. The FRB/US inflation expectations are very close to band-pass estimates of core inflation trend after 1980, but it is very different during 1960–1980 where the "data" are based on econometric estimates by Kozicki and Tinsley (2001) and thus much less reliable. Looking at both band-pass filtered data and and stability of SPF long-term inflation expectations, I impose a strong prior of treating the implicit inflation target constant at 2% from 1997 onward.

I am aware that maintaining this *assumption* is not innocuous. Reading FOMC transcripts in January 1995 and January, July 1996 [FOMC (1996a,b)] one finds discussions among comittee members, mostly J.A. Broaddus in favour of adopting some form of inflation target and Janett Yellen opposing but actively engaging in discussion with chairman Greespan about quantification of what 'price stability' means. The issue has remained unresolved at FOMC meetings, but the debate was centered around goal of "2% inflation imperfectly measured" and the measure of inflation to focus is the core PCE inflation.[6] In 2007Q4 SPF surveyed participants on issue whether they believe that FED has implicit inflation target and if so, what measure and how this value differs from their long-term inflation expectations. Only half of the respondents assumes that FED has an implicit inflation target in range 1.5–2.0 in terms of core PCE inflation.

The "inflation gap" is calculated as deviation of inflation from its implicit or explicit target is thus computed using the measured long-term inflation expectations or a 'trend inflation' using a band-pass filter of pre-specified frequencies. Fig. 3 depicts measures of core inflation (FRB-Dallas trimmed mean and CPI ex food and energy), together with FRB/US measure of long-term inlation expectations and band-pass trend at frequencies 0–32. As can be seen from the figure, the band-pass filter at business cycle periods is more volatile than data on long-term inflation expectations coming from SPF. For sample period 1980Q1–2011Q3 I calculate such frequency that matches power spectra of the survey based inflation gap and band-pass filtered one, i.e. cycles where inflation gap has most power. I identify periodicity 0–60, i.e 15 years, with risks about "optimal frequency" being lower. What remains is to check

[6] Chairman Greenspan: "Since we have now all agreed on 2 percent, my question is, what 2 percent? Let me present the issue more specifically. You can ask the question, what is the inflation rate as appropriately measured,. . . "FOMC (1996b)

Figure 3: Core inflation and ”target”



co-movement of output and inflation deviation from the target at cycles of frequency up to 15 years, which is almost twice as much than “standard” business cycle frequency often used in the literature.

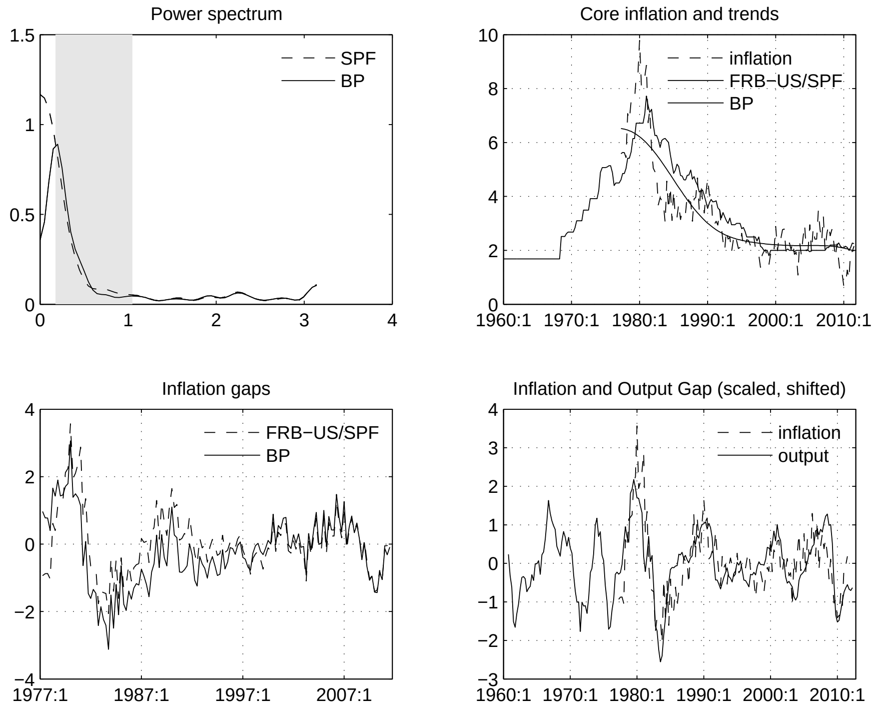
It can be seen that there is a significant co-movement between deviation of inflation from the (implicit) target and cyclical components of economic activity, if the exploration is focused on cycles strictly smaller than 60Q (15 years). This is purely statistical relationship at this stage of the analysis, though, which motivates the development of short-run Phillips curve in the sense that movements in ‘output gap’ are closely related to inflation developments, without implication if this relationship can be exploited for policymaking. From the graphs and analysis of coherence there is an intuitive result that at higher-frequencies this relationship is weak, i.e. inflation is subject to frequent occurence of short-lived shocks (cost-push) shocks, which seem not to be systematically related to cyclical downturns and booms in real economic activity.

The intuitive and simple approach to thinking about deviation from implicit inflation target, equivalent to long-term inflation expectations, is the key organizing principle of the paper for estimating NAIRU and associated unemployment and output gaps. From economic theory point of view, all what FED (or other central banks) care in terms of inflation is deviation from its target. After 1990s the credibility of monetary policy and stable inflation target make the inflation gap essentially a *observed variable,* contrary to NAIRU or potential output.

To clearly motivate three modeling aspects of inflation (target, cycle and high frequency variations) Fig. 5 depicts spectral decomposition of core CPI (ex food & energy) 1957Q1–2011Q3 using an agnostic frequency-domain filter.[7] The trend component, cyclical and high-frequency components are displayed, summing to the actual series. Dates defined by Romer and Romer (1989), dates where the authors claim the FED attempted to create recession to reduce inflation, are displayed. Incidentally, Romer & Romer episodes closely follow inflexion points in the trend component.

[7]The results come from applying an ideal band-pass filter to sample periodogram and converting the result to time domain using inverse Fourirer transform. The results using a smoothed periodogram with Hanning, Hamming or Bartlett window do not differ much, see e.g. Iacobucci and Noullez (2005). I make no claim of optimality of the filter for real-time estimation. The advantage of frequency-domain filtering is that it allows sharp dissection between frequencies.

Figure 4: Inflation and Output Gaps



Not acknowledging these three economic aspects of inflation in a model-based analysis will lead to misspecification that could mask importance of real activity cycle for inflation.

Fig. **??** depicts the cyclical component obtained using Christiano-Fitzgerald band-pass filter with the band 8–32 periods; the components are scaled to GDP gap variance, unemployment and inflation are shifted one and two quarters ahead, respectively, to account for lagged relationship.

2.3. Principal component analysis (PCA)

I find strong co-movement of real and nominal variables across selected frequencies. A static and dynamic principal component is a step further to quantify the co-movement by identifying non-structural components driving the results – eventually linked to a structural model. The important thing to check is the degree of real-nominal dichotomy, e.g. to what extent the nominal quantities (inflation) co-move with the real quantities across the frequency band.

Static PCA is carried out under very specific assumptions about the data. First, the analysis is using 'gaps' obtained by band-pass filter for real variables and the inflation deviation from a particular target. As in the previous case, the definition of the inflation measure and its target are crucial and determine the frequency band of the filter. The second decision is the choice of lag structure of the data in discrete steps (quarters). I choose the lag structure such that inflation gap enters contemporaneously, unemployment lagged by two quarters and capacity utilisation with output are lagged by three quarters. Dynamic principal component analysis (DPCA) is carried out following **?** and the results, extremely close to 6, are reported in the Appendix.

Fig. 6 summarizes the results of agnostic principal component analysis. The very important result is that a single principal component explains virtually all variance of real output, capacity utilisation and unemployment cycle jointly with sizeable portion of the US core inflation cycle.

Figure 5: Decomposition of inflation

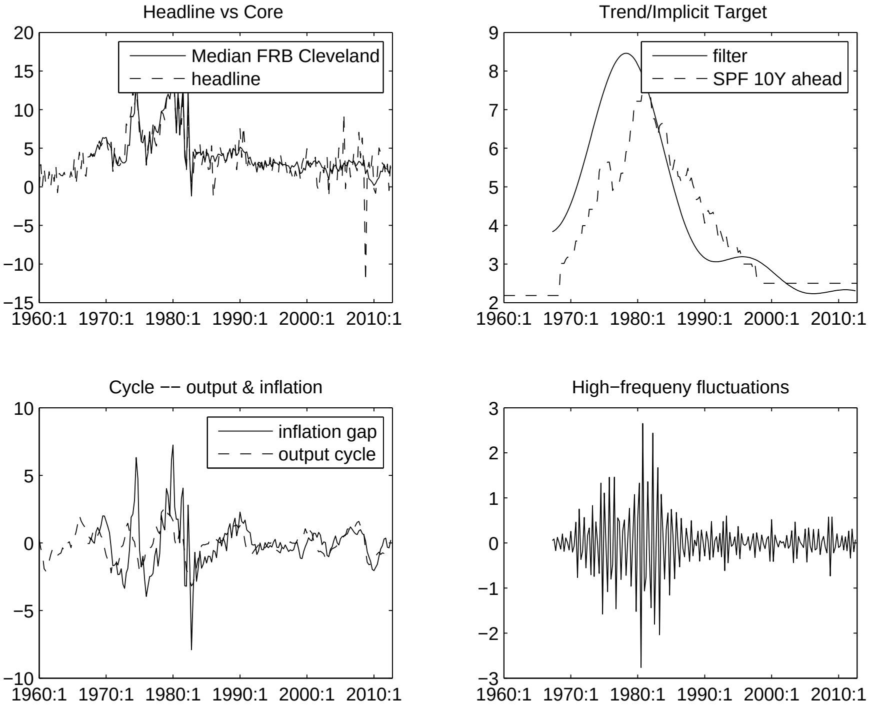
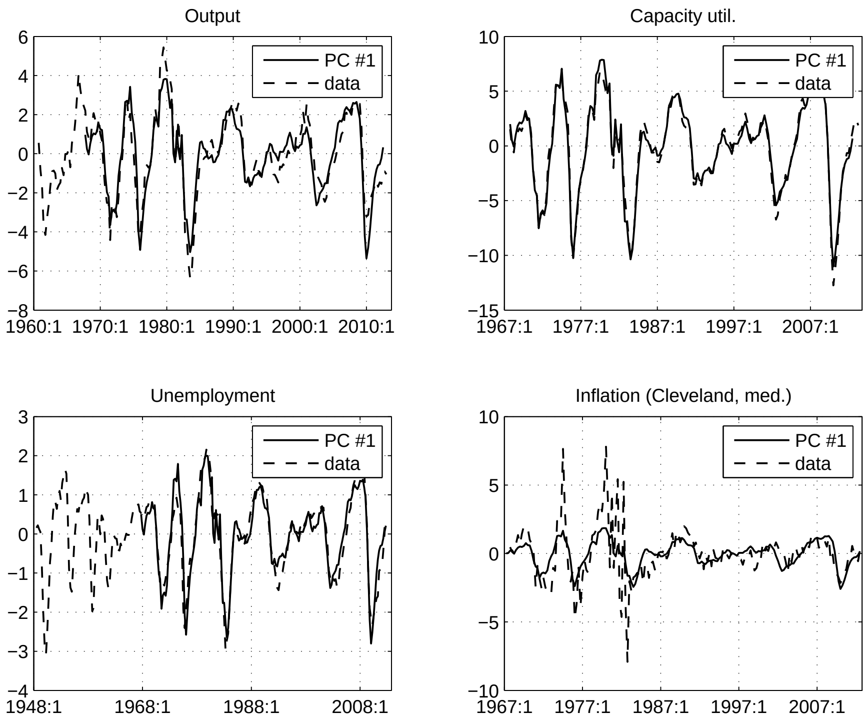


Figure 6: Principal component analysis



3. Economic Model/Filter

This section construct a simple and stylized model for the US economy to analyze relationship among inflation, output and interest rates. The model is inspired by standard one-sector closed economy Dynamic New-Keynesian (DNK) models, see e.g. Galí (2008) or Blanchard and Galí (2010). The specification of the model is very versatile and its structure is inspired by practical policy making consideration and and frequency-characteristics of the data.

Importantly, monetary policy is modeled from the point of view of modern observer, with explicit nominal anchor in the form of inflation target. Of course, Federal Reserve Board is not explicitly an inflation targeting institution and using the model to analyze history before 1990s, especially before and during Volcker disinflation, is not literally consistent with the policy viewpoints and monetary policy implemenation practices of that period. Using the assumption that there was 'implicit' inflation target, making use of long-term inflation expectations and using information implied by long-term interest rates (10Y maturity) I allow for time varying inflation target. In this paper, I do not explicitly account for lack of credibility associated with monetary policy, which was a real problem when Paul Volcker became FED chairman in 1979 – see Goodfriend and King (2005) and FOMC transcripts 1979–1985, inter alia.

3.1. Key elements of the model

Deviation from natural level of output, output gap, $\hat{y}_t$, is motivated by intertemporal decision of households and features a prominent role of real interest rate. The presence of lagged term in output gap determination is consistent with habit formation, for instance, or other type of market or expectational frictions in the economy. Via Okun's law the output gap affect unemployment gap, $\hat{u}_t$, – deviation of unemployment from non-accelerating inflation rate of unemployment (NAIRU).

Inflation, π_t is determined by short-run Phillips curve. Long-run Phillips curve is vertical – the model is consistent with arbitrary level of inflation in the long-run and thus features no long-run trade-off between output and inflation. Inflation is driven by excess demand and thus deviation of unemployment from its natural rate, or cost-push shocks. I allow for long- and short-lived cost-push (supply) shocks, with dramatically different consequences for output and interest rates – $\varepsilon_t^c, \varepsilon_t^\pi$, respectively. This allows me to obtain non-trivial cost of disinflation and inflation persistence, while having the leeway to accomodate high-frequency variations in quarter-over-quarter change in price index. Apart from expectations the Phillips curve is affected by past inflation, which can be rationalized by various indexation schemes and higher cost of reoptimizing the price than following simple rule-of-thumb rules, for instance.

Monetary policy is implemented via forward-looking interest rate rule, i_t. The rule follows inflation forecast targeting (IT), complemented with its dual mandate in the US – keeping an eye on unemployment. The FED is assumed to consider forecast of deviation of year-over-year inflation form its target three quarters ahead. The forward-looking nature of the rule is crucial and increases with degree of persistence in the economy, which interacts with monetary policy conduct. The year-over-year definition of inflation, $\pi_{y/y,t}^c$ weights down high-frequency variations in prices and creates a information delay by 5.5 months, e.g. roughly 2 quarters. This aspect of the policy precludes over-reacting to short-lived disruptions, e.g. volatile commodity prices, and captures the idea of reacting to sustained increase in prices.

As real interest rate are an important driver of the transmission, I allow for *risk premium*, rp_t on top of FED policy rates, notably in the light of recent Great Recession; I keep it exogenous for simplicity, though.[8] It is obvious that in case when aggregate demand shock $\varepsilon_t^{\hat{y}}$ and the external finance premium rp_t share the autocorrelation-properties, they are isomorphic to each other, up to a scale. This motivates also allowing for deman shocks to be autocorrelated, e.g. expectations of persistent slides in demand, tastes or financial conditions.

Apart from 3M interest rates (policy rates) I track the behavior of 1Y, 5Y and 10Y maturities, based on arbitrage expectations theory. The major driving force of long-term interest rates are changes in real interest rates, linked to potential output, and most importantly long-term inflation expectations. Yield curve facilitates to extract information on implicit inflation target in a model consistent way and provides interesting cross-check for the analysis.

[8] Although one could endogenize part of the interest rate premium (external financing premium) many ways, it seems that having an exogenous one is a reasonable start in my simple model. For instance 96% of business cycle variance of the premium is due to shock to external finance premium itself in fully-fledged DSGE model with financial frictions and banks by Christiano, Motto, and Rostagno (2010, pp. 97, Table 5b).

The critical deviation from the standard model is the treatment of trends, which is semi-structural – embracing the theory, not sticking to it completely. The model is listed bellow.[9]

The Model

$$
\begin{aligned}
\hat{y}_t &= \alpha_1 \hat{y}_{t+1} + \alpha_2 \hat{y}_{t-1} - \alpha_3(\hat{r}_t + rp_t) + \varepsilon_t^{\hat{y}} && (2)\\
\hat{\pi}_t &= \lambda_1 \hat{\pi}_{t+1} + (1-\lambda_1)\hat{\pi}_{t-1} + \widehat{rmc}_t + \varepsilon_t^{\hat{\pi}} && (3)\\
i_t &= \rho_i i_{t-1} + (1-\rho_i)[(r_{eq,t} + \pi^{\star}_{t+1}) + \iota_\pi(\pi^c_{y/y,t+3} - \pi^{\star}_{t+3}) + \iota_{\hat{y}}\hat{y}] + \varepsilon_t^{i} && (4)\\
r_t &= i_t - \pi^c_{t+1} && (5)\\
\hat{r}_t &= r_t - r_{eq,t} && (6)\\
\hat{\pi}_t &= \pi_t - \pi^{\star}_t && (7)\\
\pi^c_t &= \pi_t + \varepsilon^c_t && (8)\\
\hat{u}_t &= \alpha\hat{u}_{t+1} + (1-\alpha)\hat{u}_{t-1} + \xi\hat{y} + \varepsilon^u_t && (9)\\
\widehat{rmc}_t &= \hat{u}_t && (10)\\
i^{(N)}_t &= \tau^{(N)}_t + \frac{1}{N}\sum_{j=0}^{N-1} i_{t+j} + \varepsilon^{i(N)}_t \qquad \text{for } N = 4, 20, 40 && (11)\\
y_t &= y_{eq,t} + \hat{y}_t + \varepsilon^y_t && (12)\\
y_{eq,t} &= y_{eq,t-1} + \mu_t + \varepsilon^l_t && (13)\\
\mu_t &= \rho_\mu \mu_{t-1} + (1-\rho_\mu)\bar{\mu} + \varepsilon^\mu_t && (14)\\
r_{eq,t} &= \rho_r r_{eq,t} + (1-\rho_t)[(y_{eq,t+1} - y_{eq,t}) - \log(\beta)] + \varepsilon^r_t && (15)
\end{aligned}
$$

The trend (equilibrium) block of the model is defined in a very pragmatic way, though with an eye on theory and long-run consistency. The potential output is driven by level and growth rate shocks.[10] The important specification is that there is a central tendency towards which the potential output growth gravitates – $\bar{\mu}$. This is particularly important for estimation of unobserved shocks and interpretation of historical data, as it reduced revision properties of the model. I a priori exlude a possibility that potential ouput growth is a random walk.

The equilibrium real rate of interest in a closed economy is linked to expected growth of potential output, as would be the case in neoclassical growth model. As Goodfriend and King (2005) emphasize, the link between potential output and

The major deviation from canonical New-Keynesian model, e.g. Galí (2008), is explicit use of unemployment in the model and in the Phillips curve, using the empirical regularity of Okun's law. From heuristic point of view this is nothing new, e.g. Carabenciov, Ermolaev, Freedman, Johnson, Juillard, Kamenik, Korshunov, and Laxton (2008) use a dynamic Okun's law linking the output gap and unemployment gap using the relationship $\hat{u}_t = \rho\hat{u}_{t-1} + (1-\rho)\chi\hat{y} + \varepsilon^u_t$, which implies partial adjustment of unemployment to output gap with exponential decay of the influence of past output gaps: $\hat{u}_t = [(1-\rho)\chi/(1-\rho L)]\hat{y}_t = (1-\rho)\chi\sum_{i=0}^{\infty} y_{t-i}$. Further, the authors use high value of partial adjustment coefficient $\rho \sim 0.70$. In general, impulse response behavior of unemployment gap will result in more protracted cycles than the underlying cycle of real economic acitivity. Importantly, apart from coherence, spectral density function of unemployment and output gap will differ, hence applying an identical linear time invariant filter (e.g. band pass filter) may not identify both gaps properly. The

3.2. *Parameterization of the model*

The model is parameterized based in two steps with different emphasis on deterministic and stochastic parameters. Parameters determining steady-state are purely calibrated. I work with two set of parameters, one calibrated in a

[9] The form of the model follows essentially log-linearized version of New Keynesian model with unemployment as exposed in Blanchard and Galí (2010), with some particular deviatoins from parameter cross-restrictions. I adopt following conventions: all variables are in percents or in percentual (log) deviations, annualized.

[10] Further step in the analysis would be to allow for additional 'demand' shock on the account of misperception about the persistence of change in potential output and over-confidence as in **?**, for instance.

broad sense and the second estimated using prior restrictions. As the prior restrictions affect the estimated parameters I present model properties for both set of parameters.

The steady-state output growth of the US economy is set to 2 % per annum, reflecting demography and assumed TFP growth. In line with previous section dicussing "inflation target" and recent more direct move to inflation targeting the inflation target from 2010 onwards will be fixed at 2 %.[11] Steady-state real rate of return on riskless assets is set to 1 % (?). Calibration and estimated parameters summary is given in Tab. XY **??**. The baseline calibration was guided by empirical regularities from time and frequency domain discussed in previous section, notably relationship between inflation and GDP cyclical components. Further the interest rate response and sacrifice ratio of permanent disinflation is broadly in line with FRB US model, FRB (1996). The calibration of the semi-structural form corresponds to Calvo-Phillips curve with full indexation on past inflation and average duration of price contract of four quarters. Output dynamics features persistence and realtively smaller interest rate sensitivity, corresponding approximately to habit formation parameterization of 0.75 and elasticity of intertemporal substitution around 0.25. There is an important role of interest rate smoothing.

The estimates of parameters were obtained using a penalized maximum likelihood, technically identical to Bayesian procedures of **?**, inter alia. Yet, the priors were solicited from the baseline calibration, making use of inspection and analysis of the data sample at hand. In that respect, the priors are not independent from the data and this needs to be born in mind. The sample size is 1990:Q1–2011Q3. The identification analysis using Fisher Information Matrix evaluated at calibration, prior mode and posterior mode displays fair degree of identification. Further, contrasting prior and posterior distributions one can infer that data sample is informative in most cases about the value of the parameters.[12]

The impulse-response properties of the baseline calibration and estimated model are depicted jointly bellow for selected shocks. Shock responses are broadly in line among these two parameterizations, yet the estimated model features less persistence and more agressive monetary policy. Most notable implication is the much lower sacrifice ratio in case permanent disinflation.

4. Model properties

Model properties can be analyzed along multiple dimensions, which is what this section does.

4.1. Impulse Response Function

Permanent credible disinflation. Credible disinflation by 100bps is displayed at Fig. 9 in the Appendix. The assumption of credibility of monetary policy regime is not appropriate before Volcker disinflation and perhaps even no earlier than 1990 where first inflation targeters in New Zealand started to write a new history of monetary policy-making. The assumption is that change in inflation target is publicly announced and needs not to signalled by increase of the interest rate.[13] The expectation channel and associated increase in real interest rate depress real activity, peaking at -0.6 % output gap, nominal central bank lowers interest rate to achive a new policy neutral rate and mitigate cost of disinflations. In the model, inflation drops relatively quickly and undershoots the new inflation target. Despite credibility, there is no difference between "cold turkey" or gradualism, disinflation is costly in the model due to real and nominal persistence embodied in the economy. Further, it takes around five years to reach a new equilibrium.

The sacrifice ratio (cumulative loss of output at annual basis) is 1.2. The output loss magnitude is comparable to one obtain from FRB/US model of the FED, see (FRB, 1996, pp. 41, Fig. 7), for post-1970s policy setup of the model, both in case of model-consistent and VAR expectation formation. The simulation in FRB/US is also close in magnitudes and duration of the credible disinflation, where it takes 5.5 years to reach the new equilibrium, with sacrifice ratio of 1.3 and 1.7 in case of model consistent and VAR expectations, respectively.

[11]In its press release on Jan 25, 2012 FOMC states that "that inflation at the rate of 2 percent, as measured by the annual change in the price index for personal consumption expenditures, is most consistent over the longer run with the Federal Reserve's statutory mandate. Communicating this inflation goal clearly to the public helps keep longer-term inflation expectations firmly anchored, thereby fostering price stability and moderate long-term interest rates".

[12]Details are avilable upon request.

[13]see **?** for imperfect information problem when private sector does not observe directly if the change in policy rate are associated with change in inflation target or other shocks.

As can be seen from Fig. 9 the long-term yield jumps on impact and slightly overshoots its new long-term level. Not surprisingly, in absence of change of change in equilibrium real rate of interest the inflation expectations dominate the dynamics.

The estimated version of the model features lower sacrifice ratio... TBW.

Demand shock. An expansionary demand shock of 100bps leads to hump-shaped response of the real economic activity, pushing the consumer inflation up around 0.3 %. Monetary authority reacts by increase of policy rates such that inflation is stabilised in the monetary policy horizon of about 2.5 years.

Cost-push shock. Longer-lived cost-push shock, a negative supply shock, has persistent effects on both inflation and unemployment. A 1.5% increase of inflation reduces output by 0.25%, which is a sizeable reaction of the real economy. Although the real interest rate drops on impact it is stabilized, as well as inflation by monetary policy reaction peaking at 40 basis points.

This is not the case of short-lived (noise) cost-push shock, which results in a non-persistent one-off spike in inflation with small consequences for output, unemployment and cosequently forward-looking monetary policy reaction is negligible.

4.2. Frequency Response Function of the Model-implied Filter

The linear rational-expectations equilibrium solution implies a state-space model amenable to estimation of the unobserved, model-implied variables and analysis of the estimator.

5. Interpreting history with the model

Bellow I test the model for consistency with developments of US economy from 1960 to 2011 and carry out counterfactual simulations.

5.1. Nominal anchor found – 1985–2011

TBW

5.2. Drifting nominal anchor and the Volcker disinflation – 1954–1984

TBW

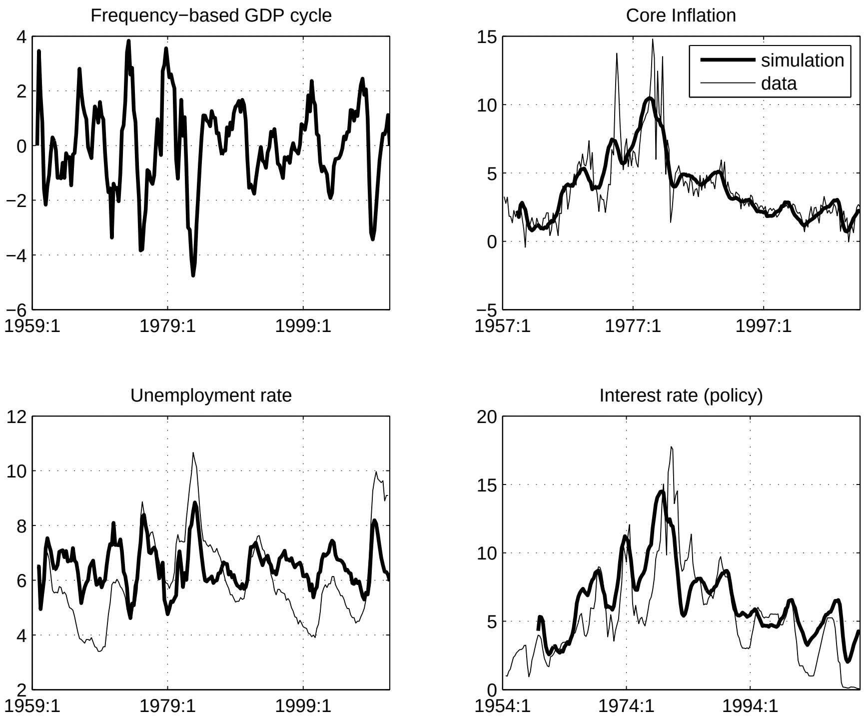
5.3. Contefractual simulations

In order to asses the calibrated model, I resort to counterfactual simulation showing the relationship between output and inflation at Fig. 7. I simulate only unanticipated demand shocks in a such a way to match the profile of frequncy-filter based cyclical component of US GDP, at frequencies $0 - -32$.

6. Identification and Observable Decompositon of the Output Gap

TBW

Figure 7: Counterfactual simulation: demand shocks only



7. Conclusion

This paper examines the state of short-run US Phillips correlation, or, understood as systematic relationship between real economic activity and inflation. The results strongly support the existence of a short-run US Phillips curve and suggest that core inflation cycles are determined mostly by demand shocks, broadly measured.

The close co-movement of output, unemployment and output argues against the real business cycle in the U.S. economy. Further, the staggering strength of co-movement between real and nominal macroeconomic variables at business cycle frequency suggest that structural models with small amount of shocks may be succesful for understanding the data, as long as the driving force resembles an aggregate demand shock in its effects. Andrle (2012) uses dynamics principal component maximum likelihood estimator for stochastically singular models exploting the stylised facts exposed in this paper.

Monetary policy plays a crucial role in the existence of the Phillips correlation and thus the Phillips curve per se. The empirical relationship embodies a response of monetary policy authority to demand disturbances, but the response does not elimate the co-movement of output and inflation altogether. A simple dynamic model, incorporating a forward-looking New Keynesian Phillips curve, which is consistent with the evidence outlined in the paper.

The identification of inflation-relevant output cycle based on observed inflation developments may have important policy implications. First, the cycles live at longer frequencies than usually assumed by business cycle theorist. Still, the inflation-relevant cycle is clearly closing following the Great Recession, which would suggest that further monetary easing would fight structural problems in the economy mostly. This view may be corroborated by recent revision of the CBO short-term NAIRU towards 6% and work by Sahin, J. Song, and Violante (2012) on the developments of the Beveridge curve.

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8. APPENDIX

8.1. Filters & Spectral Analysis

Sample spectral density. Sample spectral density is calculated using a Bartlett kernel. In case of non-stationary variables, e.g. GDP, first diferences are used to calculate cross-spectral density with other variables. For calculating the co-herence of the level of the non-stationary variable with other variables integration filter is applied, for instance:

$$\mathbf{S}_{x,y}(\lambda) = \mathbf{T}(\lambda)\mathbf{S}_{\Delta x,y}(\lambda)\mathbf{T}(\lambda)^H \qquad \mathbf{T}(\lambda) = \begin{bmatrix} \frac{1}{1-\exp(-i\lambda)} & 0 \\ 0 & 1 \end{bmatrix} \qquad 0 < \lambda \leq \pi \tag{16}$$

where the super-script H denotes a conjugate transpose.

When coherence is calculated between two series, the coherence remains unchanged if both series are pre-processed by linear time invariant invertible filters. Coherence is defined as

$$\rho^2_{x,y}(\lambda) = \frac{|S_{x,y}(\lambda)|^2}{S_x(\lambda)S_y(\lambda)} \in [0, 1] \qquad \text{for } 0 < \lambda \leq \pi, \tag{17}$$

where $S_{x,y}$ denotes the cross-spectrum of x and y. Intuitively, it is a cross-correlation of the data at particular frequencies (bands).

Observable decomposition & frequency-response function. The RE solution of the model is cast into a state-space form

$$\mathbf{Y}_t = \mathbf{Z}\mathbf{X}_t + \mathbf{D}\varepsilon_t \tag{18}$$

$$\mathbf{X}_t = \mathbf{T}\mathbf{X}_{t-1} + \mathbf{R}\varepsilon_t, \tag{19}$$

where $\mathbf{X}_t$ denotes transition and $\mathbf{Y}_t$ measurement variables. The impulse response function (IRF) of the model is

$$\mathbf{X}_t = \mathbf{B}(L)\varepsilon_t \qquad \mathbf{B}(L) = (\mathbf{I} - \mathbf{T}L)^{-1}\mathbf{R}, \tag{20}$$

which is used in a standard way to inspect response of economic variables to structural shocks and carrying out structural-shock decomposition of historical data.

To properly investigate the properties of the model and estimated structural shocks I carry out also population- and finite-sample analysis of the implied filter

$$\mathbf{X}_t = \mathbf{\Omega}(L)\mathbf{Y}_t, \qquad \mathbf{\Omega}(\lambda) = \mathbf{S}_X(\lambda)^{-1}\mathbf{S}_{X,Y}(\lambda) \tag{21}$$

8.2. Impulse Respones Functions of the Model

Figure 8: Demand shock

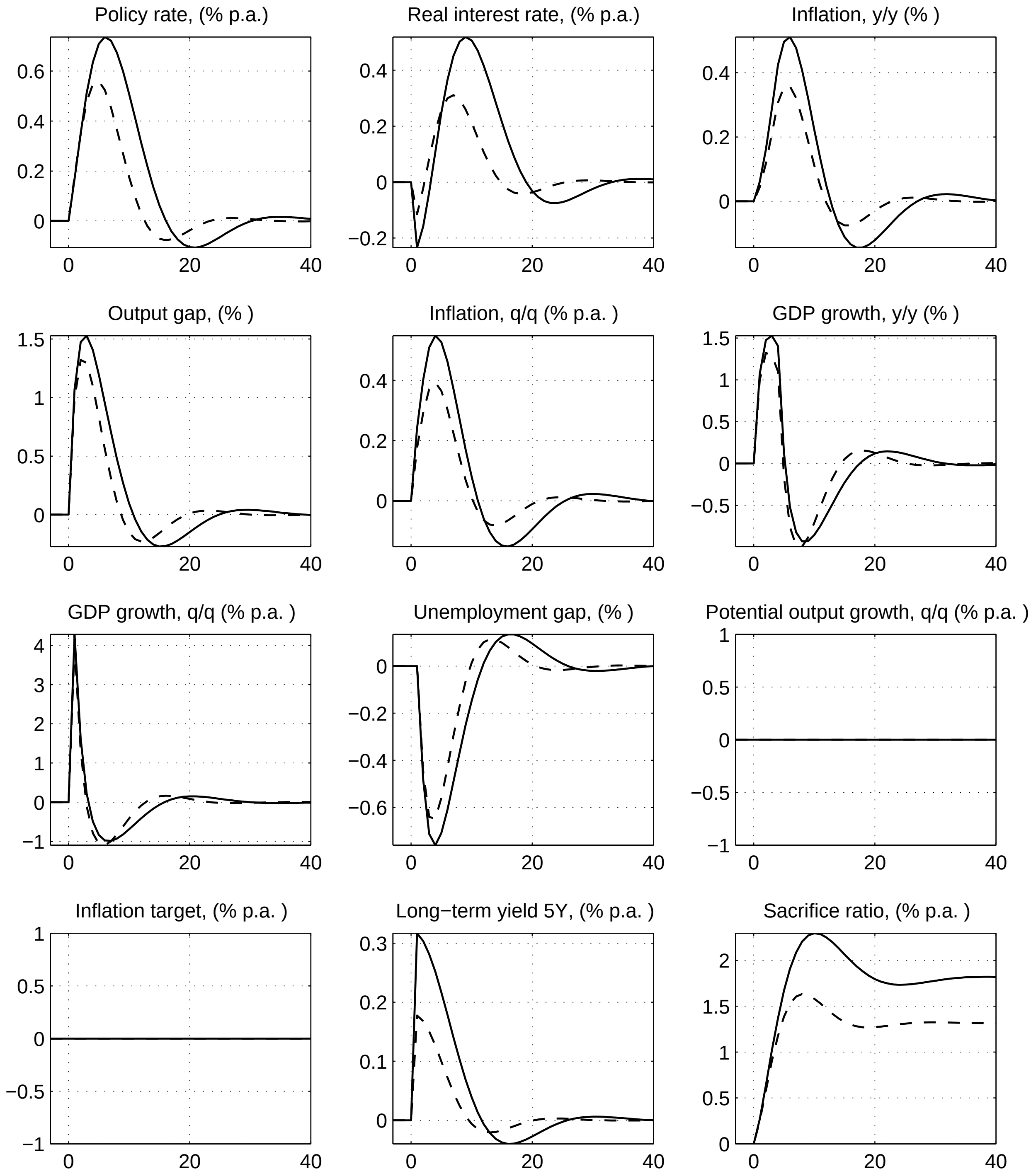


Figure 9: Credible disinflation (100bps)

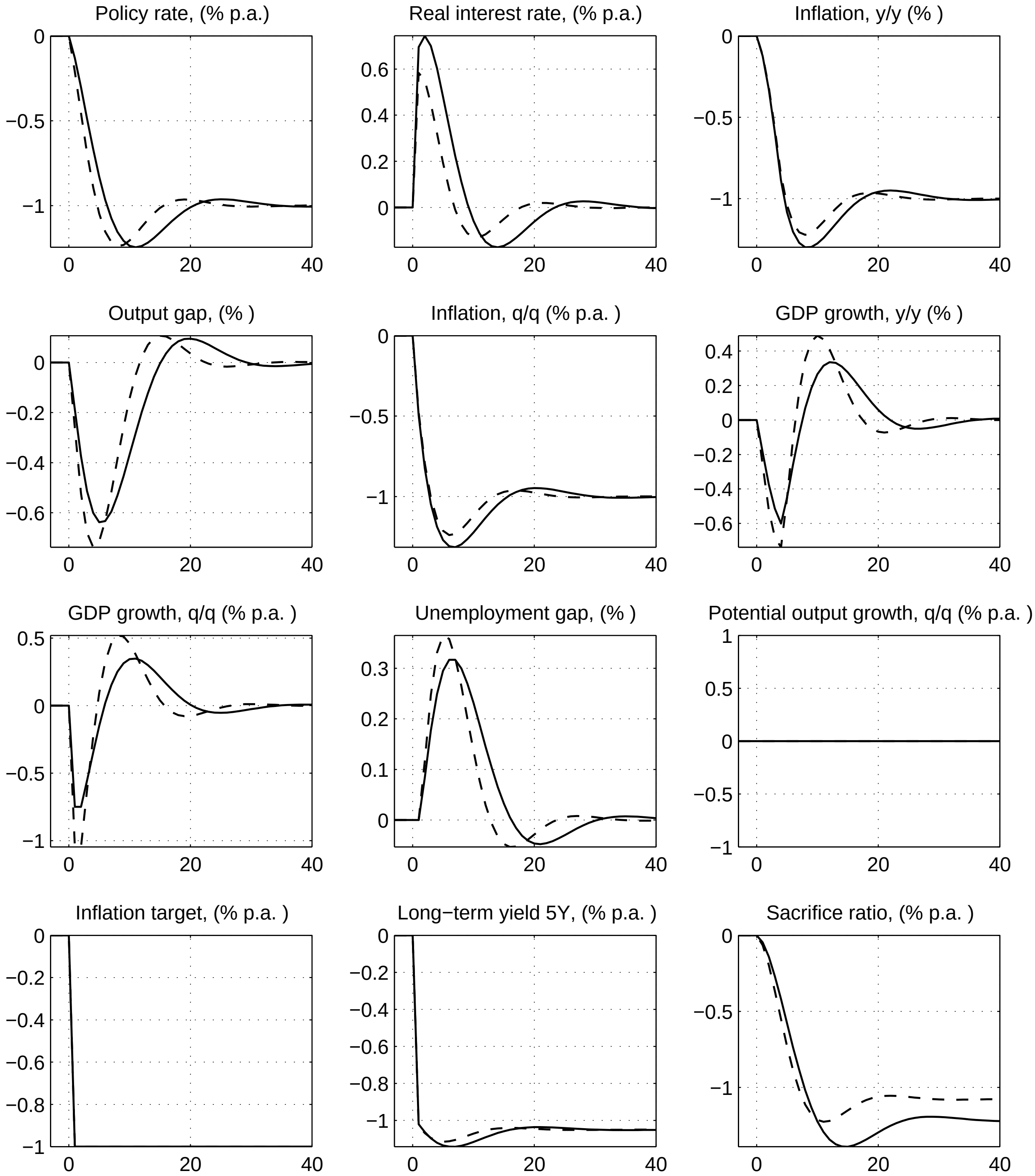


Figure 10: Long-lived cost-push (supply) shock

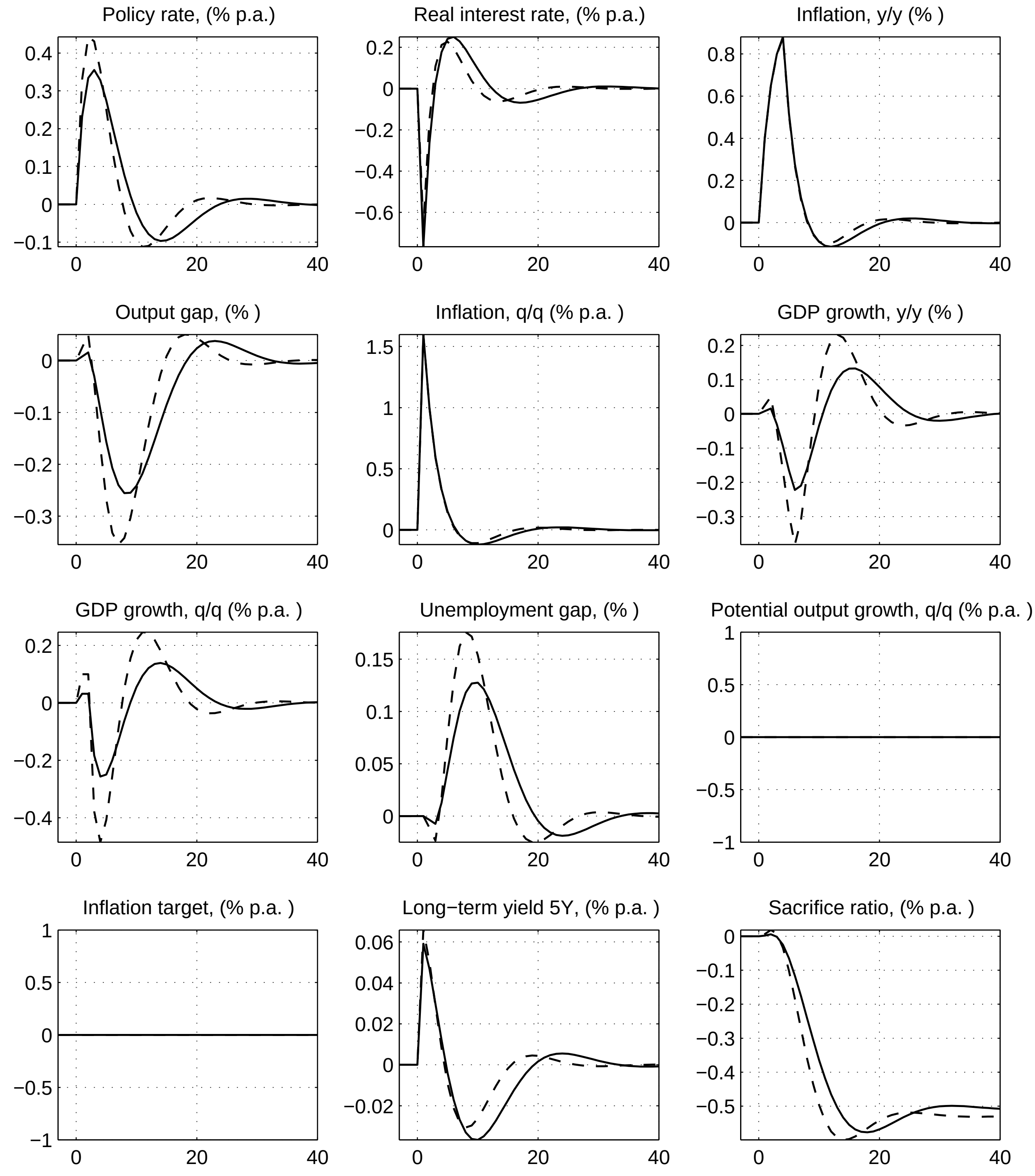
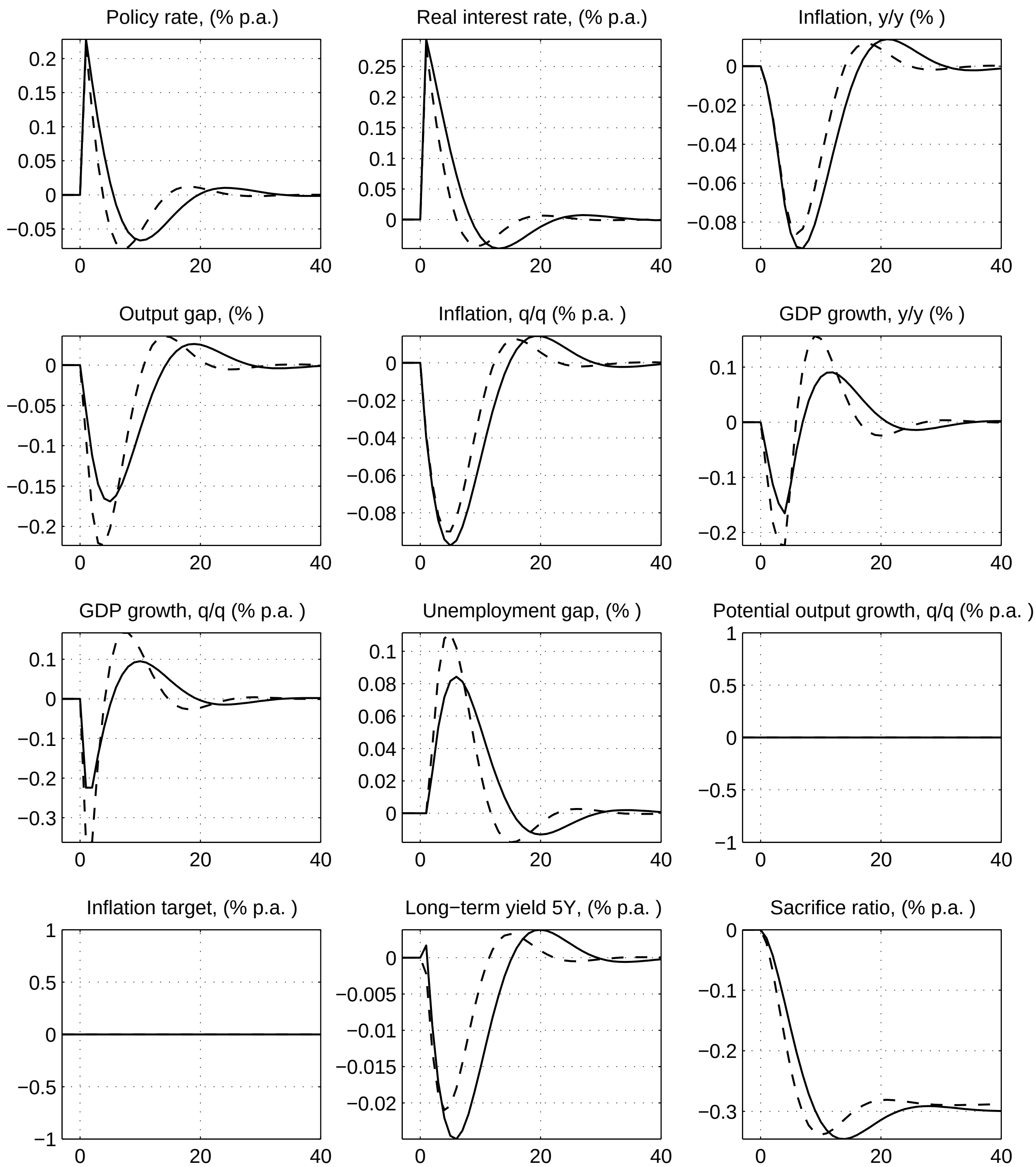


Figure 11: Monetary policy shock (25bps)



8.3. Graphs

8.3.1. Data – Sources & Definitions

Data used in the analysis are as follows:

- Real GDP –
- Unemployment –