Estimating Structural Shocks with DSGE Models*

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Contents Page

Tables

Figures

Abstract

Economists use dynamic economic models to test theories and estimate underlying structural economic shocks to interpret historical developments. An important starting point in the model formulation process is that structural shocks are uncorrelated random disturbances. The model properties – impulse response function, spectrum, or forecast error variance decomposition– implicitly assume this essential property of shocks. Yet, the actually estimated 'structural' shocks are strongly correlated as a rule rather than exception. Correlated structural shocks are a sign of misspecification. The paper proposes a new method for estimating structural shocks that are uncorrelated and makes a distinction between structural shocks and residuals. The framework also allows to deal easily with a possible stochastic singularity of the model. The paper motivates the need for uncorrelated structural shocks with a discussion of stylized facts and some common sources and symptoms of model misspecification. Misspecified models will 'fit' the data very poorly with orthogonal shocks.

Keywords: structural shocks; orthogonality; stochastic singularity; DSGE; misspecification

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I. INTRODUCTION

The key presumption of most dynamic economic models is that structural shocks should not be correlated among themselves. The assumption of uncorrelated structural shocks is always a point of departure when the models are formulated. Most of the properties of the model –impulse response function, spectral density, forecast error variance decomposition, or cross-correlation statistics – depend on this fundamental assumption. However, in the process of estimation of structural shocks using statistical techniques, like the celebrated Kalman filter, it is a rule rather than exception that the uncovered shocks are strongly correlated. Although the signs of shocks being cross-correlated can be easily seen in many empirical papers, the issue is all but ignored. This paper identifies the problem associated with correlated shocks as a model misspecification and suggest an alternative way of structural shocks estimation.

Inspecting the structural shocks is important since they allow researchers to interpret historical developments and test their theories. Yet, our models are misspecified. We do not know how many genuine 'structural' shocks are needed to validate our theoretical models, nor we are sure what these shocks are. Economists must work with misspecified models, because there are no other models. A distinction between structural shocks and residuals is thus useful. In our case it is just the structural shocks that are required to be uncorrelated, while residuals become basis for a measure of 'fit' or misspecification of the model.

The paper puts forth a novel approach for estimating structural shocks with an orthonormality constraint and weak constraints on number of shocks in the model. By restricting the shocks to be uncorrelated–with a leeway to choose a degree– the structural shocks are estimated such that the restriction is specified and the distance of the model from observed data is minimized. However, due to orthonormality restriction even in stochastically regular models the model may not be able to explain total variance of the observed data, leaving some residuals. Severely misspecified model will 'fit' the data very poorly, leading to large residuals. There is also no restriction on the number of structural shocks to be included in the model in principle with the new technique, after residuals are accepted as fact of live and part of the result. Au contraire, the analysis of residuals has always been a hallmark of misspecification tests in econometrics!

Estimating uncorrelated structural shocks helps to identify quickly the degree of model misspecification. The literature and the results of the paper suggest strong cross-correlation among macroeconomic variables in most countries, which DSGE models have difficulties to cope with. Strong commonalities also stand behind the recent success of the dynamic factor models in forecasting and explaining the data. With a misspecified structural model this easily results in correlated 'structural' shocks, which is illustrated using a New-Keynesian DSGE models following the important contributions by Justiniano, Primiceri, and Tambalotti (2010)[JPT], Christiano, Trabandt, and Walentin (2011) [CTW], or Christiano, Motto, and Rostagno (2014) [CMR], among others, and that is typical for most New Keynesian DSGE models as we discuss below. The fit of these models breaks down when the condition of uncorrelated structural shocks is applied. The models have no structural shock that would explain a strong, positive, and contemporaneous co-movement of consumption and investment, among other issues. The issue is extremely common with DSGE models, where the positive co-movement of output, consumption, investment, and hours is hard to achieve, while being present in the data. Most DSGE models are failing the 'test of the Adelmans' (Adelman and Adelman, 1959) as explained below.

In comparison with other areas of macroeconomics, the nature of structural shocks in DSGE models looks sometimes like a complete opposite. Unlike structural vector autoregressions (SVARs) or structural dynamic factor models (SDFM), the identification of structural shocks in DSGE models spurs directly from the behavioral and accounting structure of the general equilibrium model itself. This is not the case with SVARs or SDFMs, which are simply just a representation of the auto-covariance structure of the data in the first place and structural models only in the second. However, the roles get quickly changed, when a DSGE model with a priori uncorrelated shocks is applied to data, with possibly severely correlated estimated structural shocks. Factor models and SVARs, on the other hand base the very identification of structural shocks on the assumption of no correlation by factoring the covariance matrix of residuals. Without shocks being orthogonal, there would no identification or meaningful impulse-response analysis.

Computationally, the newly proposed method of shock estimation is straightforward to implement and applicable both to linear and nonlinear models. It is well known that the Kalman filter is an ingenuous recursive implementation of a least squares problem. Formally, this paper formulates the structural shocks estimation as an explicit least squares problem which is solved using the singular value decomposition (SVD) and thus easily accommodates stochastic singularity. Importantly, the least-squares problem is augmented by the penalty for correlated residuals which enables researches to vary the tightness of the assumption. The problem is close to the 'orthogonal Procrustes' problem in linear algebra, though the solution of 'Procrustes' does not apply.

II. STRUCTURAL SHOCKS... OR NOT

The importance of structural shocks being 'truly structural' is huge and the issue has been debated in the literature. Often the issue is debated jointly with the structural nature of models' parameters. Both parameters and shocks are obviously artificial constructs, a theory to be reconciled with the real world.

TBW

Should structural shocks really be uncorrelated? In this paper it is argued that they should. Not only is the assumption of uncorrelated structural shocks at the very start of the model formulation, it also is the crucial identification requirement for the shock being meaningful. If two shocks go hand in hand, then it is simply just one shocks and needs to be treated like it. A structural mechanism, a theory needs to be developed to find the actual source of variation. Orthogonality of structural shocks is the key identification principle in econometrics and in science that allows for counterfactuals and causal inference. Two variables can be systematically related either because one is causing the other, or they share a common cause, see e.g. Pearl (2009). Having correlated shocks means confounding.

After all, for many the assumption of uncorrelated shocks is simply obvious. That is why most economists start with uncorrelated shocks in the formulation of the model in the first place. Yet all this is quickly resigned upon, when the model is 'fitted' to the actual data and the structural shocks are estimated. Rarely is the covariance structure of the shocks reported or discussed. Yet, when the model can be reconciled with the data only at cost of correlated shocks, it is easy to notice in shock decomposition of observed variables and usually in variance decompositions.

Having correlated structural shocks is always a sign of misspecification but the opposite does not hold. [proof is easy. insert a proof] The greatest empirical failure would not be a failure to explain the dynamics of the data without any error, but to explain the full dynamics with severely correlated structural shocks. The sole fact that the shocks are not correlated, however, does not mean the model is a plausible explanation of the data, unless the structure and the theory are investigated further. The tests for misspecification in this paper are in some sense related to the 'test of Adelmans', see Adelman and Adelman (1959) and King and Plosser (1989). The question is – would an analyst tell the model-generated data from the actual sample? This, of course, is necessary but not a sufficient condition for the model to be the 'right' one, the useful one.

A. Stylized Facts on Co-Movement and Commonality in Macroeconomic Data

In the U.S. and other OECD countries majority of business cycle fluctuations in macroeconomic data can be ascribed to just a few sources of dynamics. The premise that only a small number of sources of fluctuations are responsible for most of the dynamics in the macroeconomic data is the raison d'être for the successful literature on dynamic factor models, see e.g. Stock and Watson (2002), Forni, Hallin, and Reichlin (2000), or **?**. Using hundreds of mostly monthly data researchers argue there are 2–6 genuine dynamic factors driving most of the macroeconomic dynamics. It is also worthy of noting that these 'factors' are orthogonal by construction. The key idea is the one of the reference cycle as originated by Burns and Mitchell (1946): macroeconomic series are intimately connected.

Recently Andrle, Brůha, and Solmaz (2014) have extended the factor analysis and demonstrated that a single dynamic factor dubbed 'demand' can explain most of the business cycle. Using quarterly data for the U.S. and other OECD countries, the authors have shown a just one dynamic principal component can explain most of the cyclical variations in real and nominal variables. Fig. 1 –borrowed from the above mentioned paper– depicts cyclical components of the U.S. selected macroeconomic variables and a portion fitted by first, first two and first three dynamic principal components since 1980s.[1] The factor is labeled as demand, since it explains both real and nominal variables well, see Fig. 2. It cannot be argued that output, consumption, investment, or unemployment do not feature a dominant and regular common cycle. Yet, that is what most DSGE models do and what becomes the reason for seriously correlated structural shocks.

For the purpose of the dynamic economic models, the definition of consumption or investment is often modified. The reason is usually to account for proper treatment of durable goods consumption or other simplifications of the model. Following Justiniano, Primiceri, and Tambalotti (2010) Fig. 3 depicts (normalized) cyclical statistics of the U.S. data where private consumption comprises services and non-durable goods only, with investment defined as gross private investment and durable consumption goods. Output, consumption, investment, and hours worked were also adjusted for population growth, see the Appendix for data source and definitions. Inflation is expressed as a GDP deflator. As can be seen from Fig. 3

[1] See the results starting from 1950's in the Appendix.

the strong co-movement of consumption, output, and investment remain to hold in the transformed data set for the whole sample from 1952:Q1 to 2013:Q4. Further, it is easy to demonstrate that these data characteristics persist for a wider frequency band than 0–32 and are not a result of 'spurious' filtering, see the Appendix for spectral analysis.[2]

The factor modeling literature also bears implications for number of structural shocks and makes a case for stochastic singularity being a virtue. For this reason solutions allowing to have a few but well-thought structural shocks are to be preferred to a proliferation of 'structural' shocks lumped into a model just to cope with stochastic singularity. Another important consequence of the analysis in Andrle, Brůha, and Solmaz (2014) is that when over 60% of co-movement among a set of variables can be explained by one factor orthogonal to others, it is like observing effects of one dominant structural shocks, to which the actual impulse-response function of the model should be benchmarked. There is no doubt there are many structural shocks operating on the margin and in specific historical episodes. The majority of fluctuations, however shares a very predictable co-movement. Have we ever seen a recession with depressed investment and consumption flat or rebounding?

The strong co-movement of macroeconomic series is obvious also for growth rates of variables, albeit more affected by high-frequency variations, see Fig. 4. The figure depicts quarterly growth rates of private consumption (without durables), investment (with durables), hours worked and output as transformed for the JPT model, normalized to the identical variance. The comovement is obvious and should alleviate possible criticism of 'prefiltering' or spurious cycles induced by band-pass filtering and spectral measures.[3] For instance, the contemporaneous correlation of investment and consumption is 0.28 and 0.38 accounting for one lag.The cross-correlation of consumption, investment, and hours needs to be explained by a common shock (factor), with constant weights even through the Great Moderation, followed by the Great Recession. The co-movement is stable, which is just great. As is clear from growth and cycles, the least coherence is at very low frequencies (trends) and very high-frequencies, see Andrle, Brůha, and Solmaz (2014) for discussion.

The results on the factor structure and strong co-movement of macro data imply that to explain the cycle means to explain the co-movement. Explaining just the dynamics of the GDP series, thus, is of little to no use unless the behavior of private consumption and invest-

[2] Note that frequency-specific correlations or 'filtered' data are simply a transformation of the raw data. If applied both to a properly specified model and to data, the result should be identical or similar. Further, 'spurious' effect of statistical filters are a fable as explained, among others, in Pedersen (2001),Kaiser and Maravall (1999), or Pollock (2014).

[3] It should be born in mind that first difference operator is also a filter; filter that mitigates trend and cyclical frequencies, greatly amplifies high-frequency noise and introduces phase-shifts.

ment is explained with the structural shocks as well. The same hold for cyclical volatility of output and hours, or unemployment. It is easy to investigate frequency-specific correlations and spectral characteristics of DSGE models. The statistical structure of dynamic principal component of the data can become an important test of coherence between the model and the data and an extension of the 'test of Adelmans' as suggested by Andrle (2012).

B. Measures of 'Fit' for Dynamic Models

What is the measure of fit of a DSGE model that helps to reveal potential misspecification of the model?

C. Misspecification – Some Sources and Symptoms

Imposing the assumption of uncorrelated structural shocks creates a distinction between those and residuals. This paper also makes a distinction between *measurement errors* and *residuals*, despite their similarity and potential to mitigate the identical problem.[4] It would not be reasonable to argue that with the restrictions on shocks to be uncorrelated the model can fit the data perfectly, after all the models are just a simplification of reality. Yet, this restriction allows not to fit the data, obtain the *residuals*, and see the model's distance from the ideal situation.

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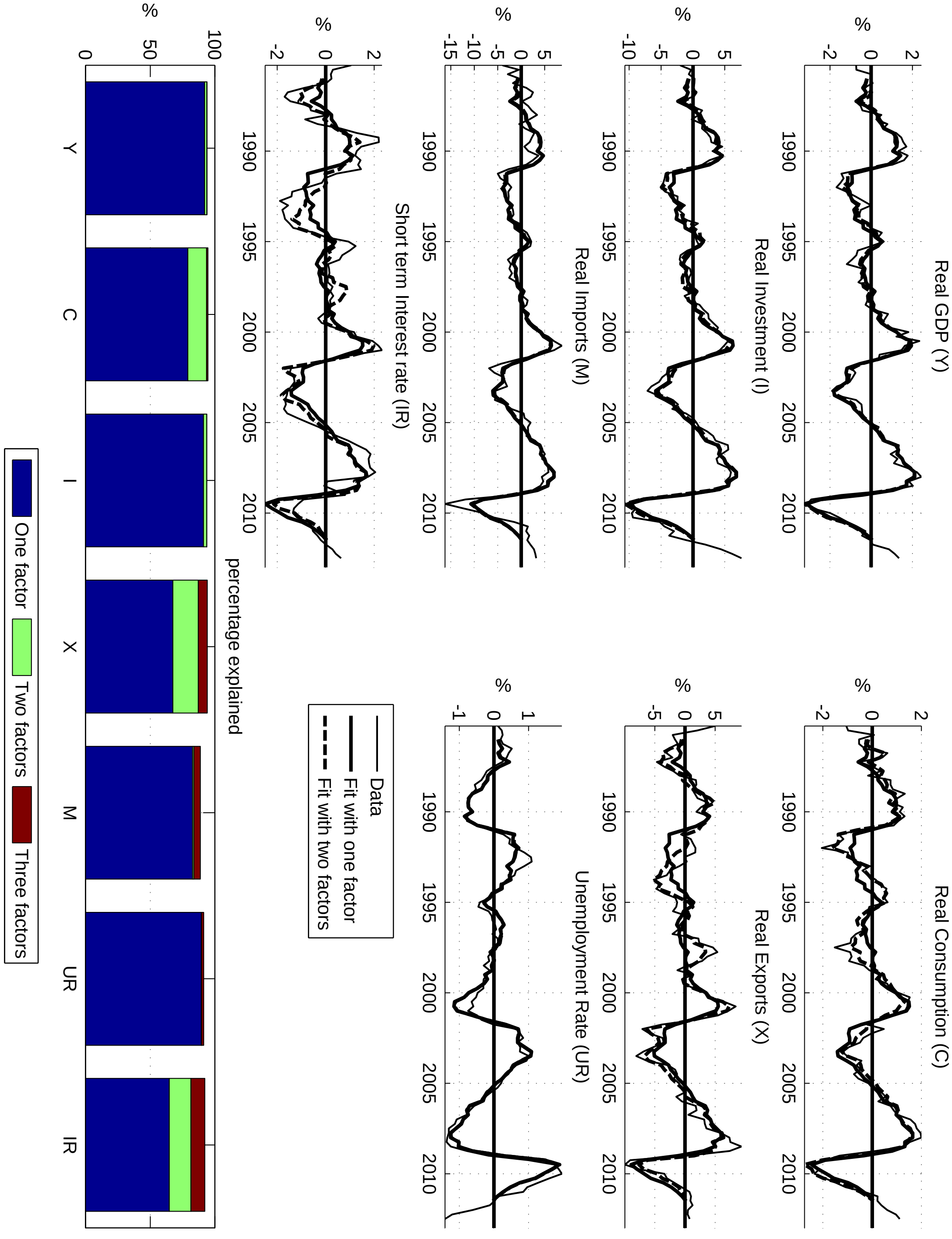
The illustration of the misspecification of explanation of sources the business cycle focuses on 'investment shocks' and 'risk shocks' in the recent literature. For instance, Justiniano, Primiceri, and Tambalotti (2010) put forth the 'investment shock', a technology shock in the production of installed capital, labeled as marginal efficiency of investment shock, as a source

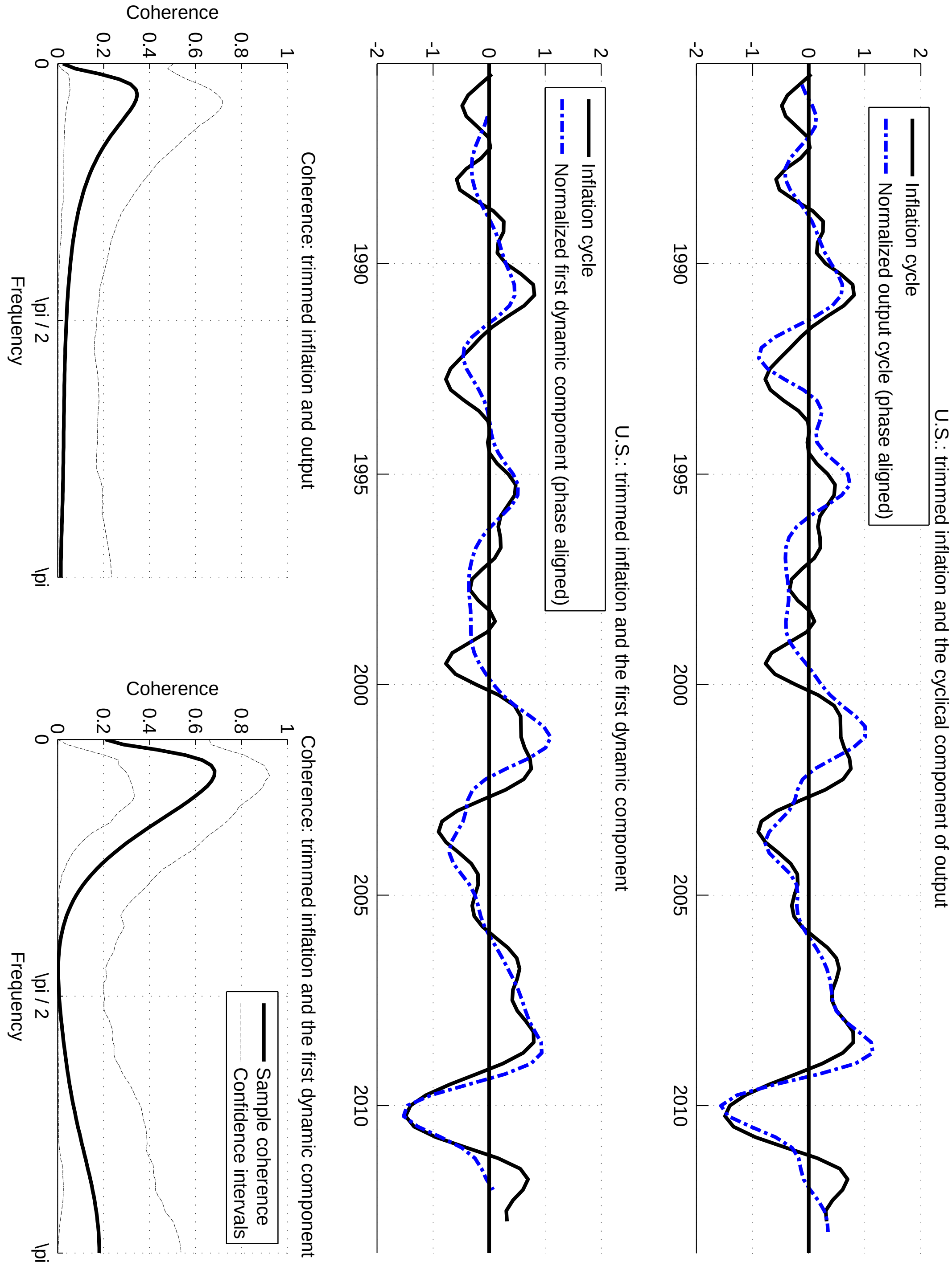
[4]First, residuals arise when the model is stochastically singular and/or structural shocks are required to be orthogonal. Second, measurement errors are a broader term used for dealing with high-frequency volatility, issues of residual seasonality, one-off measures, and importantly with actual errors of measurement. Apart from some macroeconomic data being prone to revisions, it's rarely the case that an exact counterpart of the model variable is observable in literal sense.

Figure 1. U.S. Cyclical Stylized Facts (post 1985)



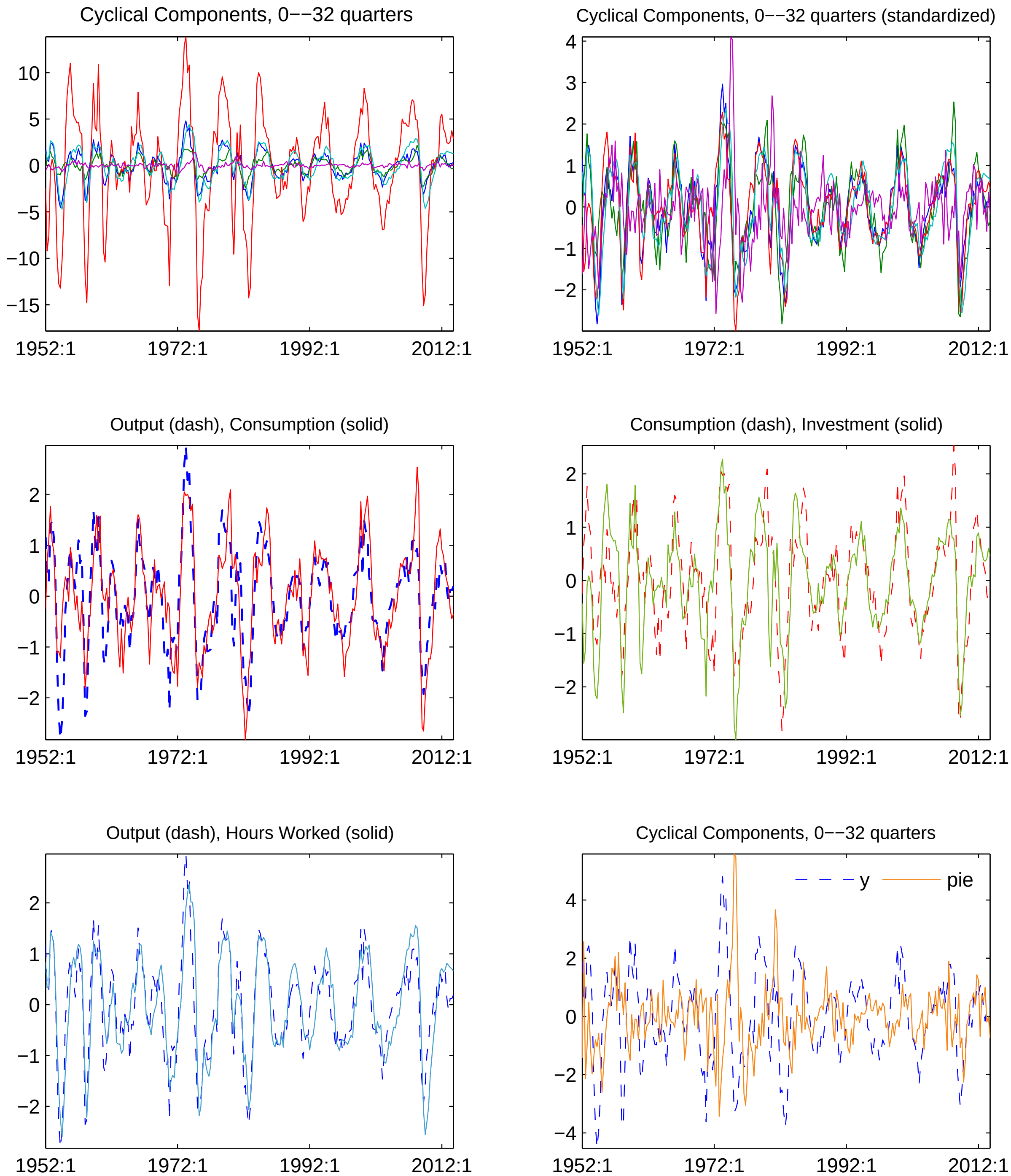
Source: Andrle, Brůha, and Solmaz (2014)

Figure 2. U.S. Nominal Cyclical Stylized Facts (post 1985)



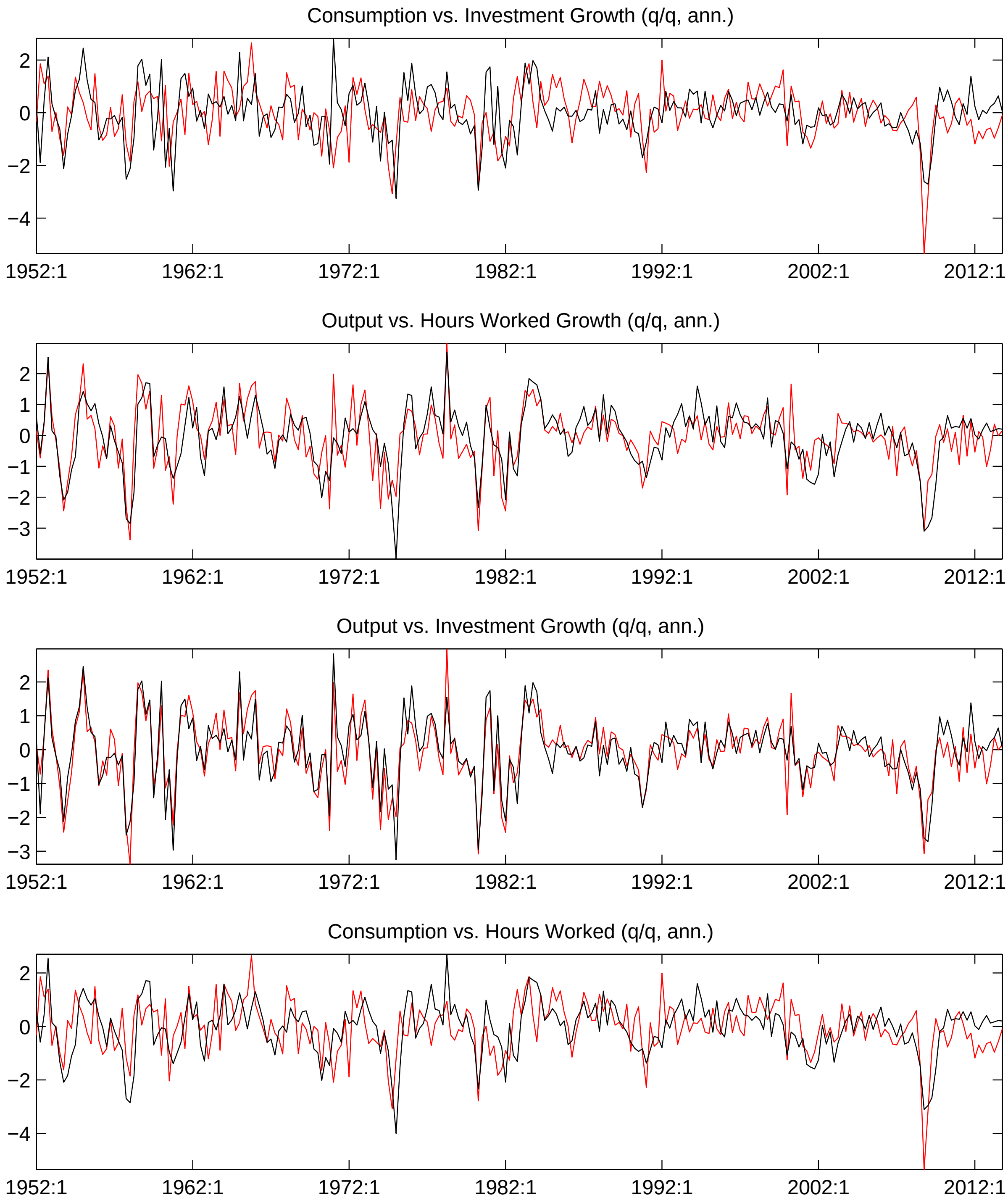
Source: Andrle, Brůha, and Solmaz (2014)

Figure 3. U.S. Cyclical Stylized Facts (JPT transformations)



Source: Haver Analytics, own calculation

Figure 4. U.S. Growth Stylized Facts (JPT transformations, normalized)



Source: Haver Analytics, own calculation of the business cycle. Literature has by and large followed their lead in relying on investment-specific shocks to explain the cycle. Recently, the literature incorporating a financial accelerator due to Bernanke and Gertler (1989); **?** (BGG henceforth) points out it's financial sector shocks and 'risk shocks' (Christiano, Motto, and Rostagno, 2014; Christiano, Trabandt, and Walentin, 2011) that are more plausible explanation of the cycle and that risk shocks dethrone the investment-specific shocks. It's argued below that neither of these shocks seems to be a realistic candidate for explaining the business cycle.

There are many cases in the literature where the misspecification is easy to spot. To the best of our knowledge, there is no paper or journal article where both a DSGE model is developed and cross-correlation of structural shocks is discussed. Yet, the signs of economic misspecification and correlated shocks are relatively easily to spot once the stylized facts above are taken into account. Let's discuss in detail how a quick look at IRFs, forecast error variance decomposition, and structural shock decompositions can raise a flag of utter misspecification.

1. Impulse-Response Functions

Not being able to find a single impulse-response function where consumption and investment are strongly positively correlated should raise a red flag. Using the stylized facts on their positive co-movement, only correlated shocks can be obtained when the model is required to explain the data dynamics fully and is misspecified. An example of this is problem is (Justiniano, Primiceri, and Tambalotti, 2010, Fig. 3, pp. 142) where investment shock boosts investment by 6%, while depressing private consumption on impact for more than one year, see Fig. 11 in the Appendix. This implies that –given the stylized facts– this shock is not a plausible explanation of the business cycle, or output for that matter.[5]

Christiano, Motto, and Rostagno (2014) argue that 'risk shocks' can replace the investment shocks after one takes into consideration financial variables. Unlike the negative short-run co-movement after a marginal efficiency of investment shock in their model, the unanticipated risk shock results into a decline of private consumption as well as of investment. The drop of consumption, however, is small and gradual on the backdrop of sharp drop in private

[5] In their online JME Appendix, pp. 15, JPT admit that the model fails 'capture the contemporaneous correlation between consumption and investment growth.' They, however claim that 'This correlation is slightly positive in the data, but essentially zero in the model'. This statement describes the model perfectly, less so the actual data they work with, see Fig. 4.

investment. In the troff of private investment response to the shock in the seventh quarter, private investment is roughly 15 times more volatile than consumption. The trouble is that something like this has not happened in the United States since 1950 and most likely ever before in absolute terms, which restricts the quantitative importance of the shock. Private investment cyclical dynamics is roughly four times more volatile than private consumption and fifteen fold difference volatility does not carry through through disaggregation to non-durables either, see Fig. 3. Even when some other shock would induce a positive co-movement of consumption and investment, the volatility problem would marginalize the importance of the shock for special and rare events of idiosyncratic investment distress. This explains that the BGG inspired 'risk' shocks are failing.[6]

In order to fit the dynamics of the data with a misspecified shocks, the shocks need to become linearly dependent. In this particular example, the strong co-movement of consumption and investment elicits that the unexplained dynamics of consumption is explained by one or more other shocks, which are linearly dependent with the investment or risk shock due to strong positive co-movement of consumption and investment. In the papers discussed the preference shock takes the role of explaining consumption dynamics, that amounts to 70% of the U.S. GDP.

2. Forecast-Error Variance Decomposition (FEVD)

As already mentioned, the forecast-error variance decomposition is pre-destined on the independence of structural shocks. With correlated shocks, the FEVD becomes very much detached from the data the model was estimated with. It still does represents the properties of the model, however, and is revealing about the empirical fitness of it. The awareness to stylised facts introduced above suggests that just a few shocks should be responsible for most of the dynamics at business cycle frequencies, while potentially many shocks are contributing on the margins to each variable to accommodate interesting historical episodes and developments.

In majority of DSGE models FEVD analysis reveals misspecification when each variable has 'its own major driver'. More precisely, what happens is that the macroeconomic variables have little co-movement and each has a different shock as a source of their dynamics. This can be again illustrated with (Justiniano, Primiceri, and Tambalotti, 2010, Table 1,

[6]Further, not that in BGG model the risk premium (spread) is endogenous. It is the risk shock that is introduced to move it in an exogenous way and it is the exogenous component that drives 95% of the spread dynamics in Christiano, Motto, and Rostagno (2014) and other DSGE models with BGG financial frictions.

pp. 138). In this case consumption ‘preference’ shock contributes 52% of variance of consumption and hardly anything else with negligible impact on investment of 2%, while investment shock contributes 82% of variance in investment and over 50% of variance in output and hours worked. Moreover, over 70% or 50% of variance in inflation and wages, respectively, is due to ‘markup’ shocks to prices and wages, while the ‘preference’ shock driving consumption contributes 0% of variance in wages and 2% of inflation dynamics. That means that development of private consumption is essentially irrelevant for price dynamics – a statement at odds with the stylized facts. While ‘demand’ is irrelevant, a neutral technology shock accounts for 14% of inflation variation, yet it is to be noted that this is due to *counter-cyclical* dynamics of inflation – a technology shocks leads to a drop in inflation.[7]

Very similar pattern of FEVD can be found in the other two papers analyzed, with the small-open economy model benefiting from the exogenous foreign shocks. In both CTW (Tab. 6, pp. 2032) and CMR (Tab. 5, pp. 50) models consumption is explained mostly by the preference shock, while investment is driven by entrepreneurial wealth and risk shocks, respectively. The model in CTW is a small open-economy model and benefits from a purely exogenous demand shock from abroad, which can give arise to positive spillover effects.

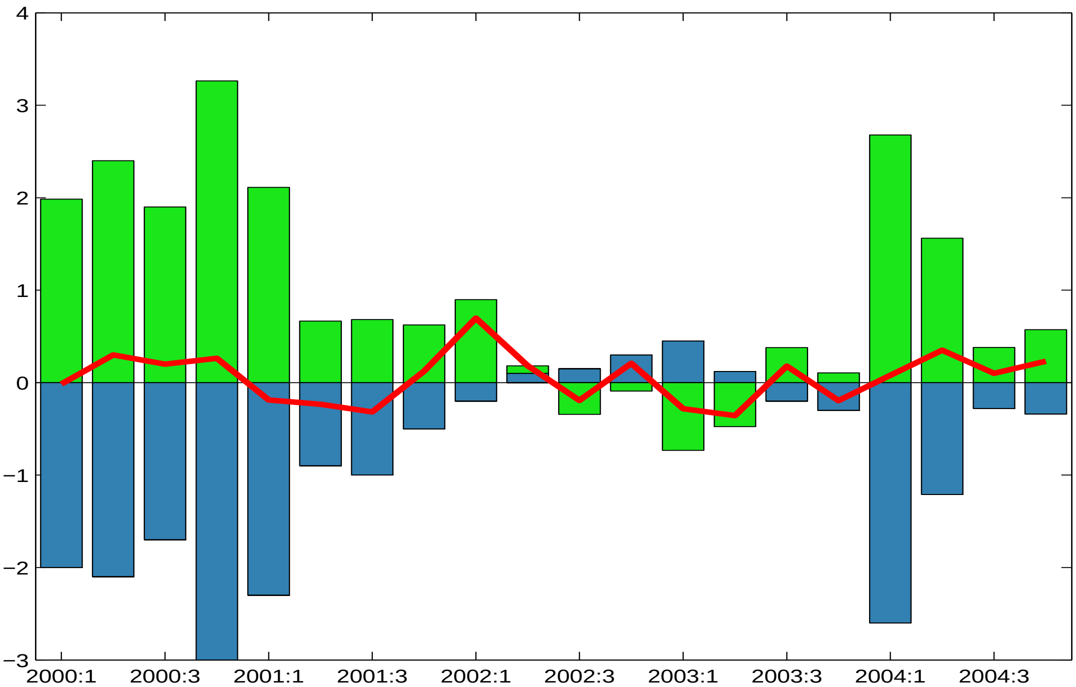
3. Shock Decompositions of Observed Data

Shock decomposition with a misspecified model often result in contribution of shocks to a variable that offset one another by a large margin. What is meant by the statement is that a 0.2% growth in a variable can be decomposed into a 10% growth due to a shock A and -9.8% growth due to a shock B in most of the periods. A graphical rendering of such shock decomposition of the actual data looks like a very colorful fish and the graph is fishy indeed.[8] The offsetting of shocks emerges when a shock creates an opposite correlation of at least two variables than what is observed in the data. Another pattern would be that each variable is explained by its shock (and hence its own color in the graph) in the limit. The spurious correlation is the more frequent case, however, usually for consumption vs. investment, or output vs. hours worked.

[7] This may not be the case for a permanent TFP shock when the wealth effect is significant. Even in this case, however, the amplitude of inflation change is too small in proportion to change in output or its cycle.

[8] Obviously, guilty as charged! As many other people, the author has produced a great deal of such artwork in the past, yet to his credit it was to his utter dissatisfaction.

Figure 5. An Example of a 'Fish' Shock Decomposition



Relatively large offsetting contributions of shocks can be found in Christiano, Trabandt, and Walentin (2011) for all reported variables, namely for inflation. The shock decomposition of output, inflation, interest rate spread, and unemployment in Figures 10–13 in CTW demonstrate a significant degree of misspecification where one shock is offsetting another. In this paper it is not argued, that at times structural shocks counteract one another, namely as economic policy is involved. In the Great Recession, for sure a monetary policy shock representing implicit tightening can work against an adverse shock triggering the recession. Yet, the shock decomposition of inflation by CTW seems relatively stable in the degree of shocks offsetting, as is the case with unemployment.

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D. Consequences

III. State Estimation with Singular Models and Uncorrelated Shocks

A. Model

It is assumed that the model can be expressed as a linear state-space model. The discussion below focuses on linear case with constant coefficients only. The model can be written as:

$$
\begin{aligned}
Y_t &= ZX_t + H\varepsilon_t, & (1)\\
X_t &= TX_{t-1} + R\varepsilon_t \qquad \text{with} \qquad \varepsilon_t \sim N(0,I); & (2)
\end{aligned}
$$

where the model is said to be *stochastically singular* if the number of shocks, $n_e = \dim(\varepsilon_t)$ is smaller than the number of observed variables, $n_y = \dim(Y_t)$.

The estimation of the state variables and shocks would normally proceed with the Kalman smoother. In the case when $n_e < n_y$, this standard solution is not feasible, unless the observed data have exactly the dynamic rank equal to n_e. Such situation is rare, since the real data are rarely rank-deficient exactly. Even with when the data had a factor structure, some idiosyncratic random noise or high-frequency variation is always present and we will assume that the process $\{Y_t^{obs}\}$ has a stochastic rank equal to n_y.

A solution to deal with stochastically singular models has been analyzed before in Andrle (2012), based on principal components. When static or –preferably dynamic– principal component analysis effectively processes the data due to strong commonalities, the model's measurement equation can be rotated to a (dynamic) principal component sub-space of the data. Thus having just one, or two, shocks with multiple observables means one needs to use one, or two (dynamic) principal components and map the measurement equation to these. This paper introduces an alternative but complementary solution, which exploits the stochastic singularity of the model itself, not of the data.

B. Least-Squares State Estimation

The estimation of structural shocks below makes use of an equivalence between least-squares, Wiener-Kolmogorov, and Kalman filtering. After all, the very nature of the Kalman smoother is that it is a recursive and efficient version of the least squares, see Kalman (1960), Whittle (1983), or Durbin and Koopman (2001). Although the time-domain and frequency-domain approaches are equivalent, each can be more elegant and easy to use in different tasks, see e.g. Whittle (1983), Gomez (1999), or Andrle (2012), among others.

Recently, in the economics literature some authors have re-stated the fact that state estimation is a least-squares problem. Schmitt-Grohé and Uribe (2010) note that this form of state estimation can be used for evaluating the likelihood function with pre-filtered data and model. That, however, can be achieved more efficiently in the frequency domain using the Whittle likelihood, see e.g. Harvey (1991), Christiano and Vigfusson (2003), or Andrle (2012). Further, Kollman (2013) that a stacked-time least squares problem can be formulate to estimate shocks in the linear state-space model. It was exactly this kind of large-scale inverse least-squares problem, however, that Kalman (1960) ingenuously reformulated in a recursive way and made computationally more efficient, or feasible at all at that time.

To estimate the structural shocks and initial conditions, the problem is cast into a stacked-time least-squares problem to accommodate rank-deficiency and restrictions on shocks. This intuitive and simple setup, however, is computationally inefficient, unlike its frequency-domain implementation discussed below. The optimization task associated with the model can be easily written as follows:

$$\min_{X_0,\{\varepsilon\}} \quad \Lambda = X_0 P^{-1} X_0 \quad + \quad \sum_{t=1}^{N} [Y_t - ZX_t]\,(HH')^{-1}\,[Y_t - ZX_t]' \tag{3}$$

$$+ \quad \sum_{t=1}^{N} [X_t - TX_{t-1}]\,(RR')^{-1}\,[X_t - TX_{t-1}]' . \tag{4}$$

It is useful to rewrite the least-squares problem in a stacked form and as a function of the initial state X_0 and stochastic shocks, ε_t, only. Denoting $\mathbf{Y} = [Y_1\ Y_2\ \ldots\ Y_N]'$, $\mathbf{E} = [\varepsilon_1\ \varepsilon_2\ \ldots\ \varepsilon_N]$, and $\mathbf{Z} = [X_0\ \mathbf{E}]$, the least-squares problem is stated as follows:

$$\mathbf{Z} = \text{argmin}\ ||\text{vec}\mathbf{Y} - \mathbf{A} \times \text{vec}\,\mathbf{Z}||, \tag{5}$$

where the 'multiplier' matrix $\mathbf{A}$ is clearly given by the structure of the model and the values of T, Z, R and H. The dimension of vec$\mathbf{Y}$ is $(n_y N) \times (n_e N + n_x)$. Further, it is trivial to see that the structure of **A** is

$$\mathbf{A} = \begin{bmatrix} \mathbf{ZT} & | & \mathbf{ZR}+\mathbf{H} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{ZT}^2 & | & \mathbf{ZTR} & \mathbf{ZR}+\mathbf{H} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{ZT}^3 & | & \mathbf{ZT}^2\mathbf{R} & \mathbf{ZTR} & \mathbf{ZR}+\mathbf{H} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & | & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{ZT}^N & | & \mathbf{ZT}^{N-1}\mathbf{R} & \mathbf{ZT}^{N-2}\mathbf{R} & \dots & \dots & \dots & \mathbf{ZR}+\mathbf{H} \end{bmatrix} = [\mathcal{O}\,\mathcal{H}]. \tag{6}$$

The solution of least-squares problem is achieved using the singular value decomposition (SVD) of the multiplier matrix A. The solution using the SVD, see e.g. Strang (2009), or Golub and van Loan (1996) for details, is numerically efficient, allows for stochastically singular models, and provides insights into conditioning of the estimation problem. In general, the multiplier matrix **A** is not square and invertible, once the initial conditions are being estimated, or the number of shocks, state variables and the sample size take rather special values. When the number of shocks is less than equal to the number of observed variables, $n_e \geq n_y$, SVD is just a standard unique solution of the least squares and the model explains all variations in the data. The solution is obviously equivalent to the one obtained with the Kalman smoother.

Unlike with the standard Kalman smoother, the solution of stochastically singular models with $n_e < n_y$ is feasible in this case. When $n_y > n_e$, the least-square problem is under-determined and standard solutions do not apply. Fortunately, this is just a standard and well-understood problem in linear algebra, when the null-space of the multiplier matrix **A** is unraveled using the SVD. The solution is not unique. It is chosen such that the energy of the shocks is the smallest among solutions available. Since the problem is under-determined, the 'fitted' values of observables do not equal to observed data, unless in the very rare case of stochastically singular input data. Hence, there will be *residuals,* errors. These residuals are, of course, quite different from structural shocks. Note that the exact null-space of the model transfer function is used, all structural equations are respected.

The solution written in terms of the SVD transformation of the multiplier matrix A is revealing of the structure of the problem. Namely, one can analyze the conditioning of **A** by inspecting its singular values. The solution requires an SVD of an $m \times n$ multiplier matrix **A** as follows

$$\mathbf{A} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1' \\ \mathbf{V}_2' \end{bmatrix} = \sum_{i=1}^{r} s_i u_i v_i'. \tag{7}$$

where $r = \text{rank}(\mathbf{A})$, $\mathbf{U}'\mathbf{U} = \mathbf{I}$, $\mathbf{V}'\mathbf{V} = \mathbf{I}$ and $\mathbf{S}_1$ is diagonal, $\mathbf{S}_1 = \text{diag}(s_1, \ldots, s_p)$, where $p = \min\{m, n\}$. When the matrix $\mathbf{A}$ is singular, then rank of $\mathbf{A}$ is r, then $s_{r+1} = \cdots = s_p = 0$.

The solution $\text{vec}\,\mathbf{Z}$ is then obtained as

$$\begin{aligned} \text{vec}\,\mathbf{Z} &= \mathbf{V}_1\mathbf{S}_1^{-1}\mathbf{U}'_1 \times \text{vec}\,\mathbf{Y} && (8)\\ &= \sum_{i=1}^{r} \frac{u_i' \times \text{vec}\,\mathbf{Y}}{s_i} v_i. && (9) \end{aligned}$$

For stochastically singular models an extra care needs to be devoted to an estimate of the state variance. For these models, the covariance matrix P in (3) is singular and does not posses an inverse. In such case the algorithm uses a regularized inverse, with the covariance matrix augmented for a diffuse component, i.e. $\bar{P} = P + \gamma I$, where I is and identity matrix.

C. Testing and Estimation with Uncorrelated Structural Shocks

Estimation of structural shocks requires further restriction, namely that the structural shocks are uncorrelated. With the restriction of uncorrelated structural shock is applied to the problem (5), the solution may not–and usually will not–explain all the variation in the observed data even when the model is not stochastically singular. Again, there will be residuals. The relevant optimization problem is given as follows:

$$\mathbf{Z}_\lambda = \text{argmin}\,\{||\text{vec}\mathbf{Y} - \mathbf{A} \times \text{vec}\,\mathbf{Z}|| + \lambda ||N^{-1}\mathbf{E}' \times \mathbf{E} - \mathbf{I}||_{\mathcal{M}}\}, \qquad (10)$$

where $\mathbf{Z} = [X_0\ \mathbf{E}]$ as defined above and $\lambda \in [0, \infty)$ is a parameter that sets the relative weight on the constraint requiring the structural shocks to be 'uncorrelated'. Uncorrelated means that the spectrum of shocks is flat, not only that contemporaneous correlation is zero. The norm $||.||_{\mathcal{M}}$ is a 'matrix norm' which could entail a variety of norms (Frobenius, Schatten, or entry-wise). Orthogonal Procrustes problems use the Frobenius norm, computations below use an entry-wise norm, which is and L_2 norm with vectorized matrices.

In this paper, numerical methods are used to solve the restricted problem. The initial conditions are obtained from solving (5) using SVD both in stochastically regular and singular case, which are then updated numerically for given value of λ. Despite the problem being close in spirit to the "orthogonal Procrustes problem", which is easily solved using the SVD[9],

[9] The problem is to find Q such that $||A - BQ||_F$ and $QQ' = I$, given known A, B. See e.g. (Golub and van Loan, 1996, pp. 601).

the particular problem does not seem to have an analytical solution. That is also true in general for the weighted orthogonal Procrustes problem, see e.g. .

Finite sample considerations are easily taken on board and should provide more realistic setup of the misspecification test and of a penalty function. In short samples, the covariance matrix of course is not an identity matrix and it needs to be tested for no correlation. The solution is to consider the sample size N and to sample $\varepsilon_{s|N} = N(0, I)$ for $s = 1, \ldots, S$ replications, which enables to use finite-sample distribution of elements in the covariance matrix or the Frobenius norm as a 'prior' distribution for the penalty, rescaled by λ. For short samples available to macroeconomists the distribution can be relatively large, see Fig. 10. There is also a well-known test for sphericity by Bartlett (1954) and tests for detecting multicollinearity based on spectral decomposition of the covariance matrix.

A more refined approach takes into account explicitly the identification of the shocks using the Kalman filter. The test samples $\varepsilon_{s|N} = N(0, I)$ for $s = 1, \ldots, S$ with the sample size N, uses the model to obtain a set of observed variables $Y_{s|N}$, and subsequently runs the Kalman smoother to obtain $\hat{\varepsilon}_{s|N}$ to be used for construction of the distribution of cross-covariance matrix and other statistics under the hypothesis of well-specified model.

The finite-sample distribution of random innovations lend itself naturally for a misspecification testing. The importance of short-sample consideration is large in the specification of the test. The test becomes a "necessary hurdle that any business cycle model must clear," King and Plosser (1989). It is, however, only a necessary not sufficient condition for the model to a plausible explanation of the data. For instance, while real business cycle model may seem like a plausible explanation in this sense, after including inflation, stock prices or other variables into set of observables, the hypothesis may break down.

1. More Refined Criteria for Identification of Structural Shocks

The restriction on shock correlation is far from perfect and represents just a first step in the analysis of model misspecification. A restriction of pair-wise cross-correlation does not address all issues of linear dependency of the structural shocks (static rank) or their temporal cross-dependencies (dynamic rank). It is often the case that the misspecification gets introduced into temporal cross-correlation among residuals.

D. More Efficient Implementation

There can be more efficient implementation of the filter, explicitly framed within Wiener-Kolmogorov filtering tradition. The implementation is explicitly frequency-domain based and makes clear what frequencies are being used for shock identification.

The frequency-domain analogue to the state-space model **??** implies the following spectral density of the measurement process $S_Y(\lambda)$ and states, $S_X(\lambda)$. The structural shocks are iid white noise with a flat spectrum by assumption.

TBW...

IV. Applications

A. "Investment Shocks" as Explanation of the Business Cycle

This section investigates a typical New-Keynesian DSGE model of and demonstrates that the main result is due to a model misspecification and produces correlated shocks. In a prominent paper, Justiniano, Primiceri, and Tambalotti (2010) have constructed and estimated a DSGE model to conclude that the "main finding is that investment shocks–shocks to the marginal efficiency of investment– are the main drivers of movements in hours, output and investment over the cycle," (JPT, pp.144). This is very true in their model, as well as that the investment shocks explain for instance less than 10% of consumption variance, see their Table 1 on pp. 138. However, in the United States the private consumption amounts to roughly 70% of GDP and is almost perfectly correlated with investment at business cycle frequencies since at least 1950. Consumption is thus explained by 'its own' shock to preferences of households, which adds almost anything to explaining volatility of investment or any other variable, for what it matters.

The explanation of the authors' findings is that the model and the role of investment shocks are misspecified and features cross-correlated 'structural' shocks. The preference shock explains consumption, investment shock explains investment and since in the U.S. data consumption and investment are strongly correlated, the estimated shocks inevitably must be strongly correlated as well. In such a situation shocks in DSGE models should not be considered structural. The model 'fits' the data only at cost of correlated shocks, where essentially each variable is driven by 'its own' shocks. For instance, around 70% of inflation dynamics is driven by 'markup' shocks, with only 8% being explained by consumption-preference and investment shocks, which drive most of the consumption and investment, and thus output dynamics. The model has virtually no role for a 'Phillips curve' where aggregate demand would be affecting inflation, which is clearly at odds with the literature, see e.g. .

The New-Keynesian model with investment shocks is parameterized using the median posterior estimates by JPT and the focus here is solely on the estimation of structural shocks. The model is taken as given, including the Bayesian likelihood estimates, no attempts to modify or extend the model are made. The model is formulated in a state-space form (1)-(2), with structural shocks transformed to multivariate standard Normal distribution. The data range from 1954:Q3 to 2004:Q4 for the baseline estimate and 1954:Q3 to 2013:Q4 for an update. The estimated 'structural' shocks are tested for uncorrelatedness using the sphericity test by Bartlett (1954) and using the exact short-sample distribution introduced above.

An inspection of structural shocks estimated using the Kalman smoother reveals a significant cross-correlation patterns. Due to the attempt to match the full data dynamics with the model, the estimated innovations feature a great deal of high-frequency variation. It is this high-frequency variation which can 'mask' contemporaneous and temporal the cross-correlation at cyclical frequencies. Even a naked eye can tell there is a co-movement among selected shocks, as depicted at Fig. 6. It is thus not surprising that despite a contemporaneous correlation of investment-specific shock with preference shock is just -0.005, when a lag is accounted for one finds a cross-correlation of 0.28, which –given the sample size and finite-sample distribution of shocks at Fig. 10– is extremely unlikely to be a mistaken for no correlation. Similar findings hold, for instance, for wage and price mark-up shocks with a contermporaneous correlation of -0.216.

To asses comovement among shocks in finer detail, measures of coherence were calculated. Coherence is a frequency-domain analogue of cross-correlation, expressed across frequency bands. It corrects for lead/lag relationship also. Bivariate coherences in Fig. 7 were obtained using a VAR model with three lags with structural shocks, omitting policy and government spending shocks.[10] The business cycle frequencies, $4-32$ quarters, are shaded in gray. The peak coherences range from 0.2 to 0.4 and usually concentrate at lower frequencies, except the case of wage and price markup and investment-specific shock with TFP (as weel as low frequencies). Considering the fact that these are coherences for 'structural shocks' that should have a flat spectrum with no coherence, the values are large. An omen of misspecification.

How does the shock estimates and 'fit' of the model change with restriction on contemporaneous correlation of shocks? When structural shocks are estimated using the least squares augmented only for the restriction that contemporaneous cross-covariance matrix of structural shocks is diagonal with unitary variance, the shock estimates do not change significantly. Unlike in (10) the constraint is now $||N^{-1}\mathbf{E}\times\mathbf{E}'-\mathbf{I}||_{\mathcal{M}}\}$, the covariance matrix is $n_e \times n_e$. In this case finite-sample considerations are ignored and $\lambda = 10000$. Modest changes are visible only for the nominal interest rate series and hours worked in 1980s, see Fig. 12 in the Appendix.

[10] The results are robust to change in order of the VAR. Parametric estimate has been chosen, but non-parametric results using the Bartlett kernel are comparable. Andrle, Brůha, and Solmaz (2014) discuss the relationship between these two estimation approaches in greater detail.

Figure 6. Estimated shocks (1954:3–2004:4, unrestricted, normalized)

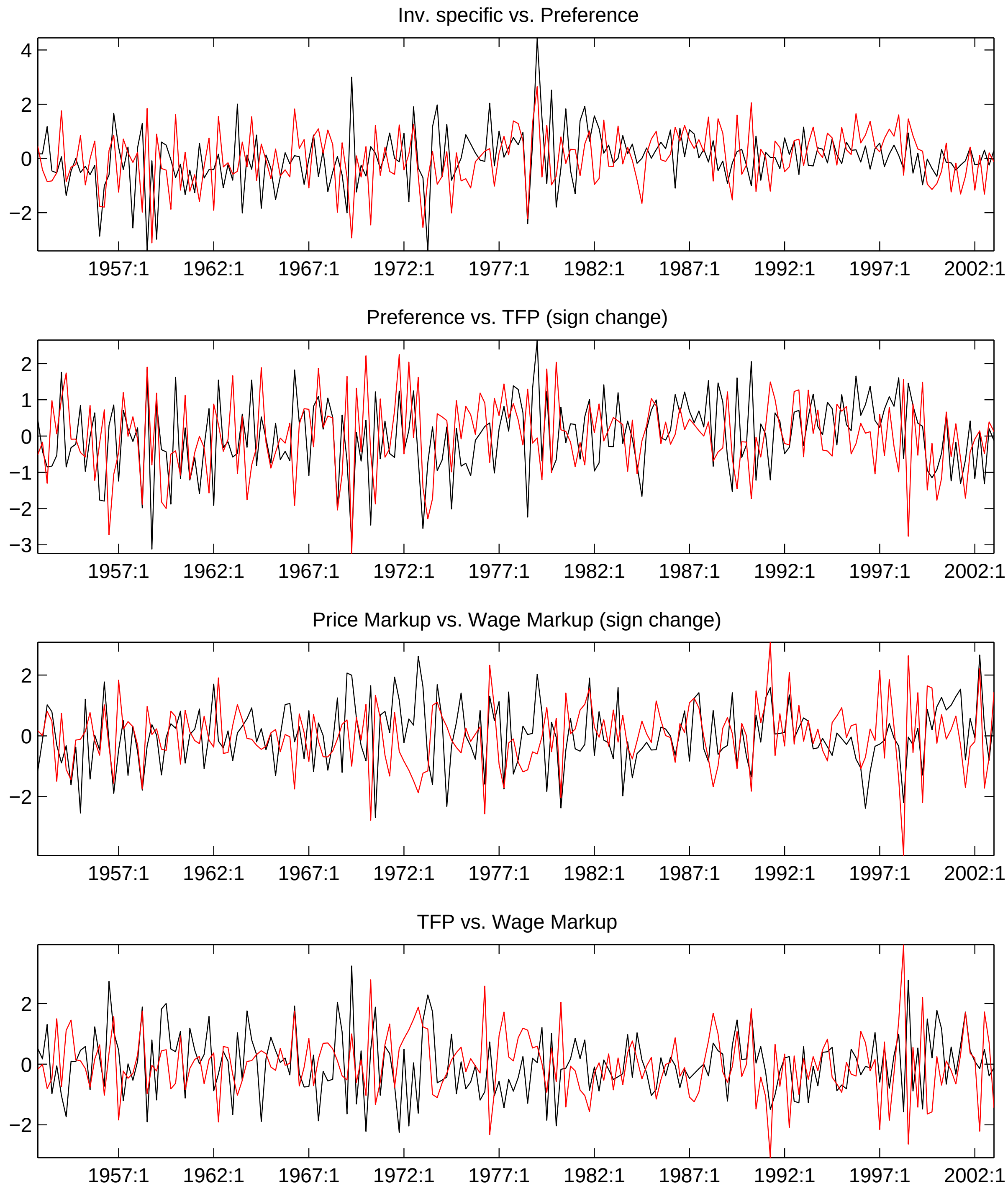
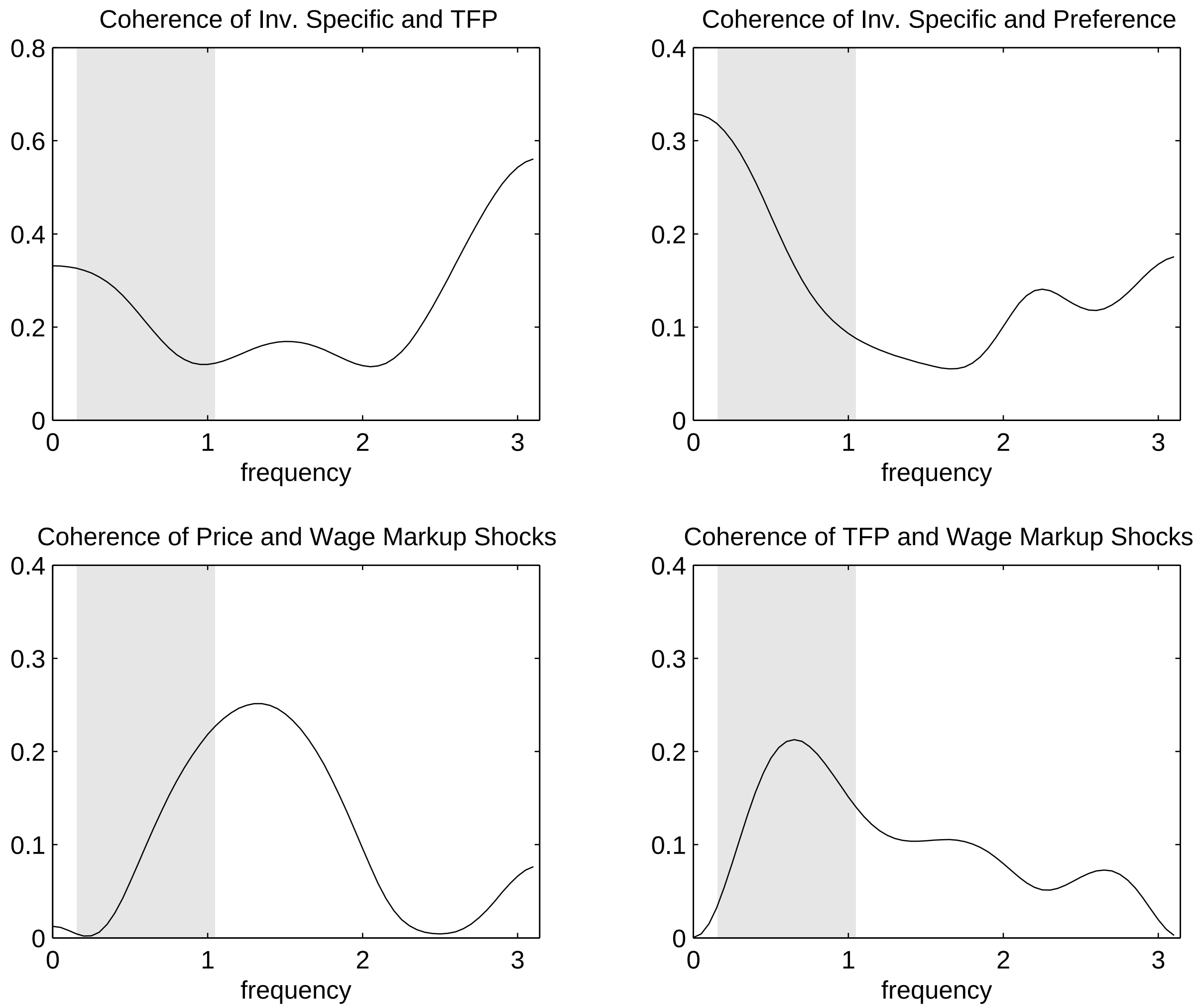


Figure 7. Coherence of Structural Shocks (1954:3–2004:4, unrestricted, normalized)



The misspecification of the model can be illustrated by its imperfect fit when structural shocks are required to be temporally cross-correlated. The model with the parameterization as estimated in the paper is used to estimate structural shocks using the restricted least-squares estimator introduced above. As the number of shocks, seven, equals the number of observed variables, the potential of the estimator to handle reduced-rank estimates is not relevant and there is always a solution of the unrestricted least-squares Kalman problem. Fig. **??** demonstrates that the 'fit' of the model is rather low in this case as the structure of the model is at odds with the data.

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V. CONCLUSION

Once estimation of structural shocks is taken very seriously, the requirement of them being uncorrelated is a very natural one. This paper introduces a new approach of estimating structural shocks in DSGE models with an arbitrary number of shocks that are uncorrelated, and a residual-based test of misspecification. The models can thus be stochastically singular. Once only uncorrelated shocks are believed to be structural, an intuitive distinction between structural shocks and residuals opens up. Hence, the structure and size of residuals give arise to a measure of the model misspecification and 'fit'. Cross-correlation among 'structural' shocks is always a sign of model misspecification and to great extent invalidates the further analysis of the models' impulse-response function, spectral density, forecast-error variance decompositions, or use for economic policy.

The importance of uncorrelated structural shocks is exemplified using a New-Keynesian model with a prominent role of 'investment shocks', and it is demonstrated to be misspecified. When the estimated shocks cannot be strongly correlated, the model does fit the U.S. macroeconomic data imperfectly. In this very case, this is due to inability of the model to explain co-movement of investment and consumption. In other models, the case is often the inability to match co-movement of output and employment, or output and inflation.

Structural shock estimation using restricted least squares is feasible in principle for both linear and nonlinear models. Without the restriction of uncorrelatedness of structural shocks, the least-squares problem has a simple analytical solution. In the case of stochastically singular models, the under-determined least-squares problem is solved by an application of the singular value decomposition (SVD), the 'queen of linear algebra'. The restricted problem is in the spirit of 'orthogonal Procrustes' problem but for the specification at hand no analytical solution has been found yet and refinement of the computational aspect is left for further research.

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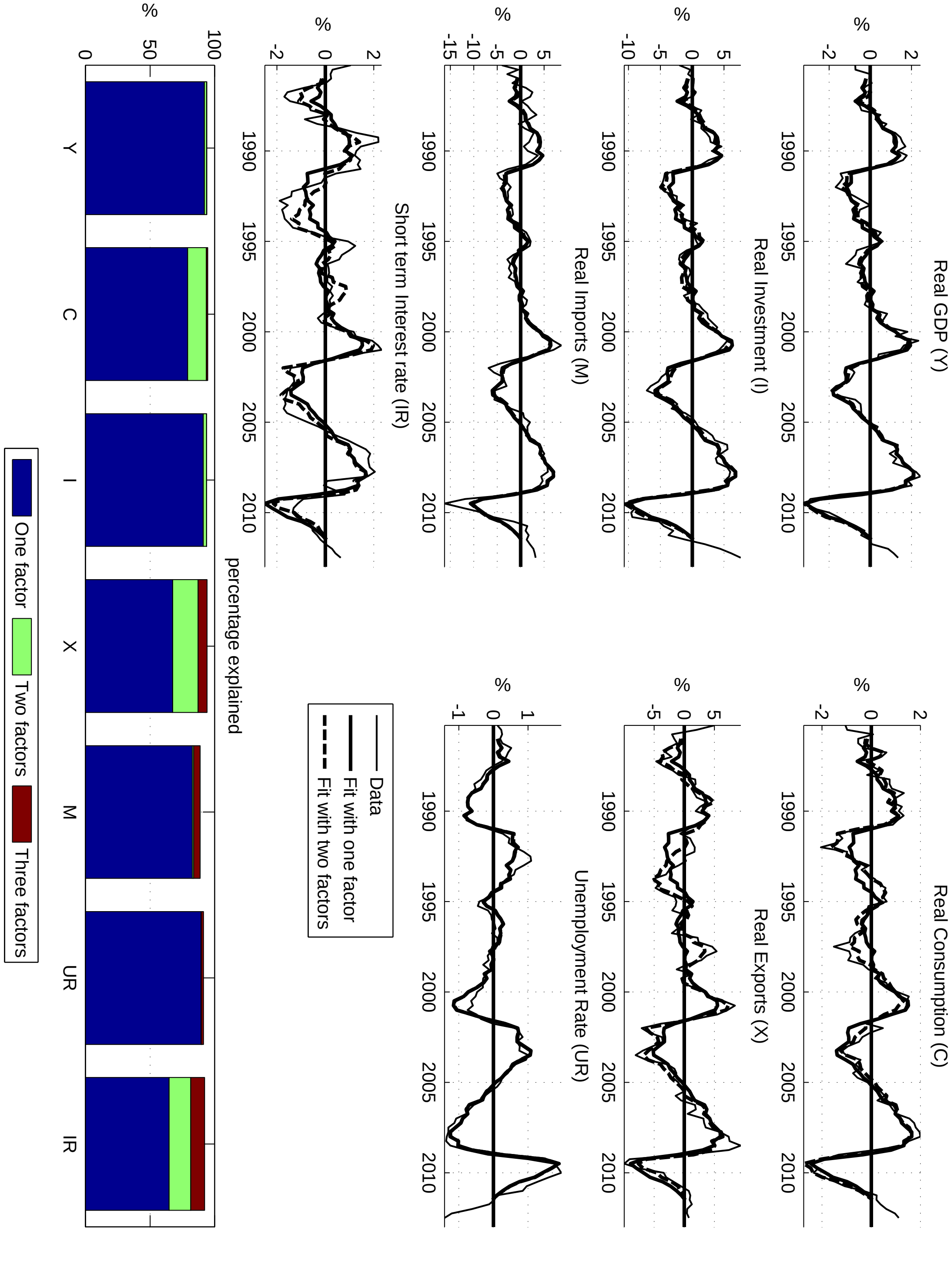
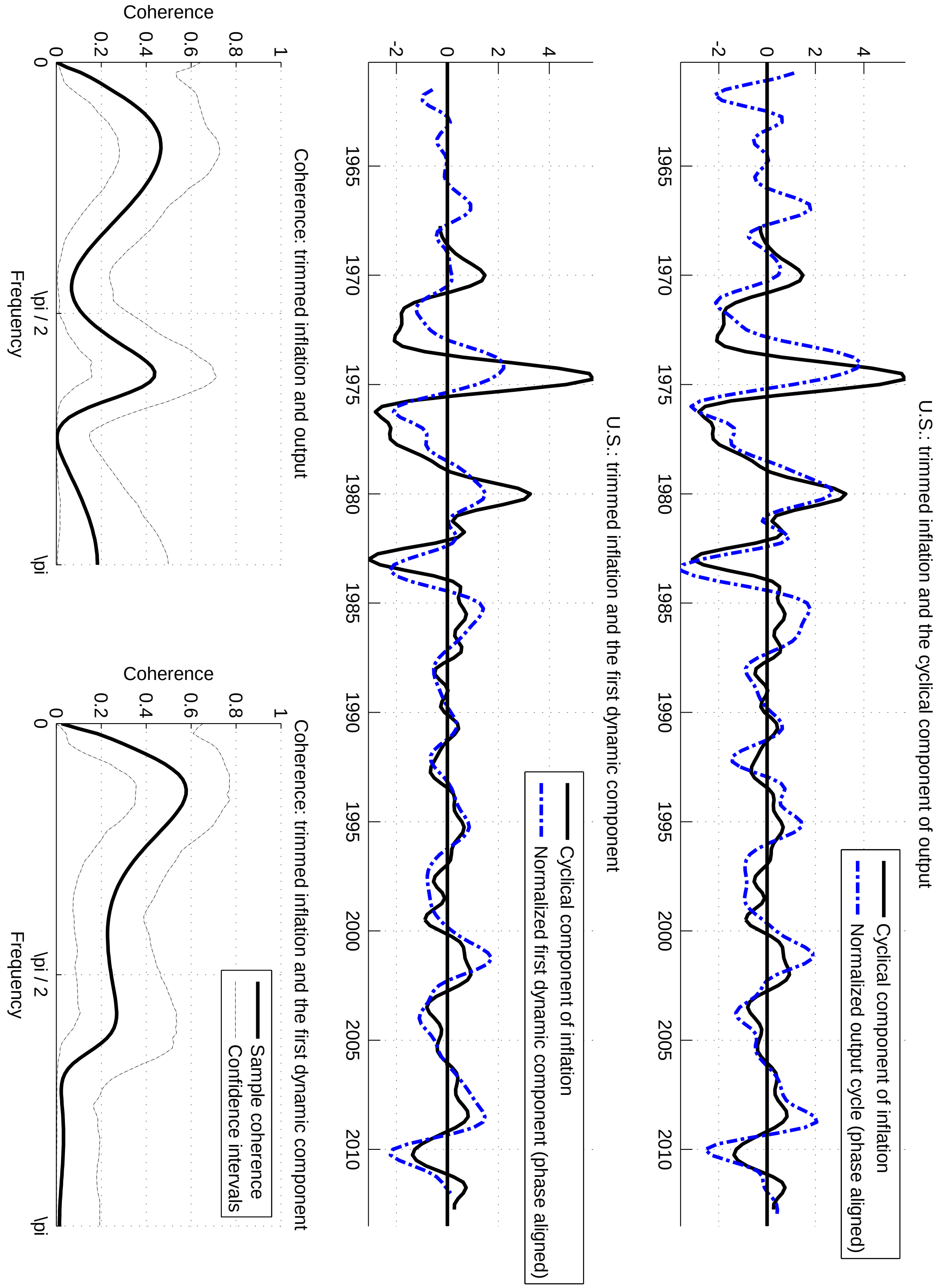
VI. Appendix: Additional Graphs

Figure 8. U.S. Cyclical Stylized Facts (post 1985)



Source: Andrle, Brůha, and Solmaz (2014)

Figure 9. U.S. Cyclical Stylized Facts (post 1985)



Source: Andrle, Brůha, and Solmaz (2014)

Figure 10. Distribution of Covariance Matrix Elements for $N(0, I)$

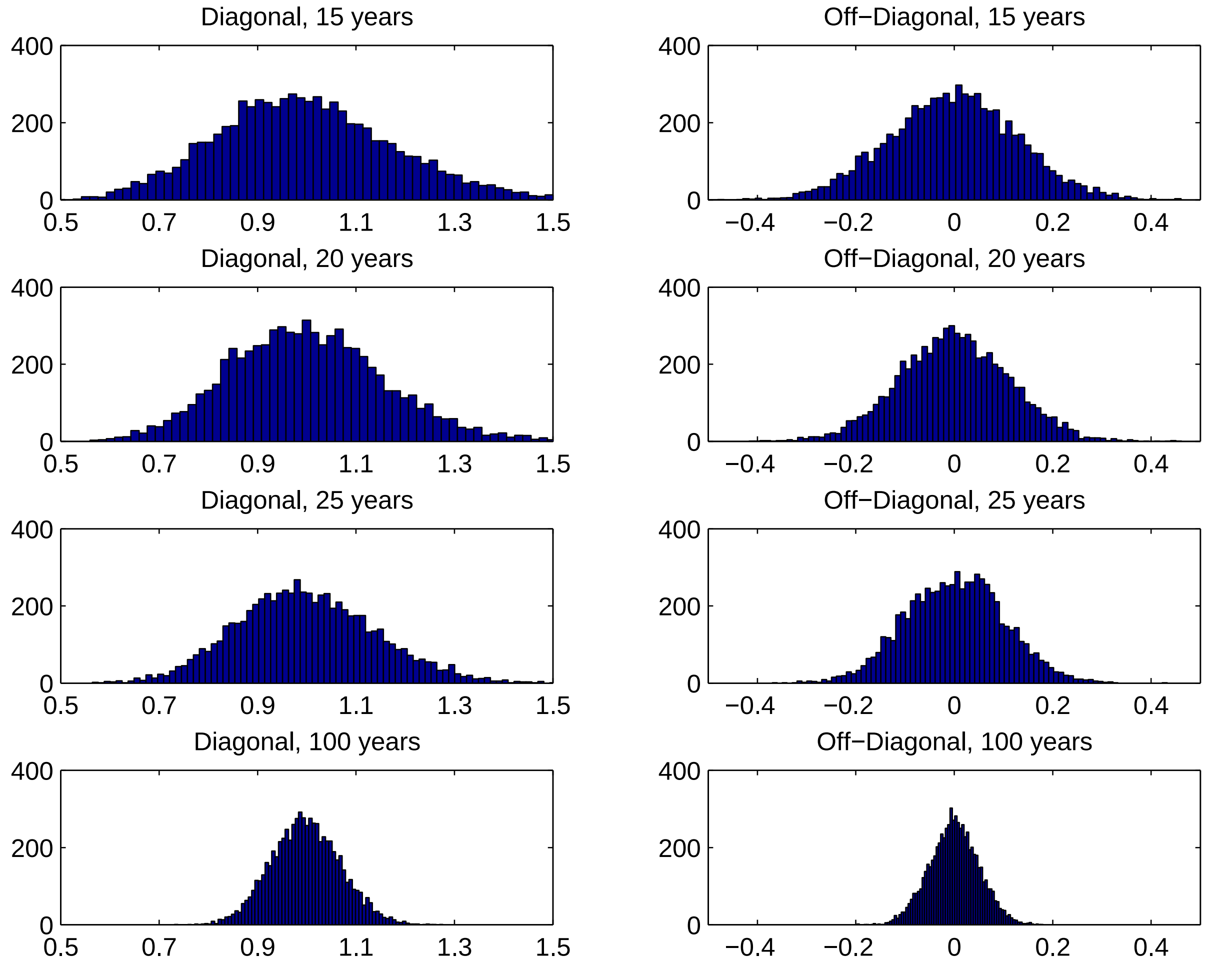


Figure 11. Response to Investment Specific Shock in JPT model

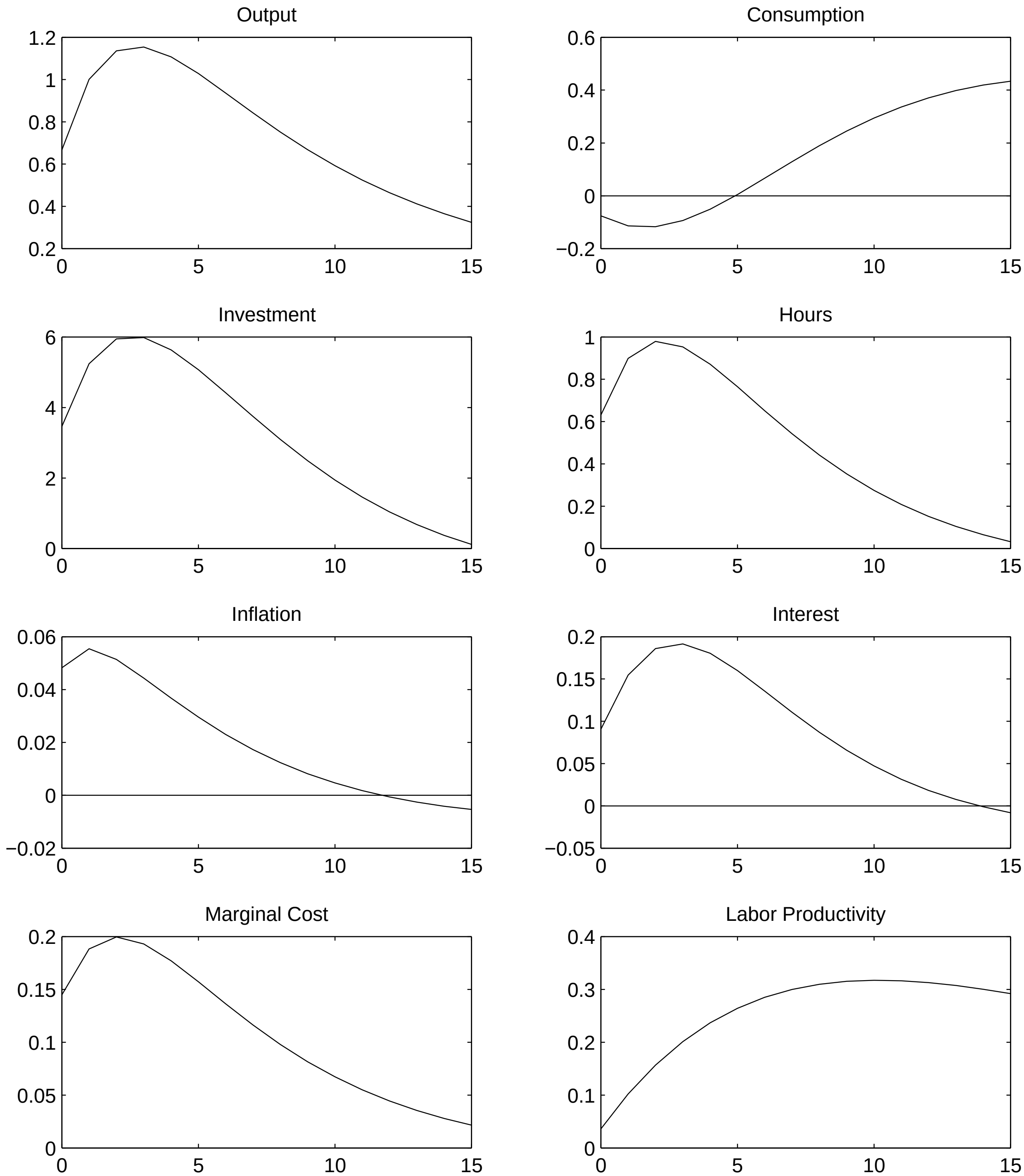
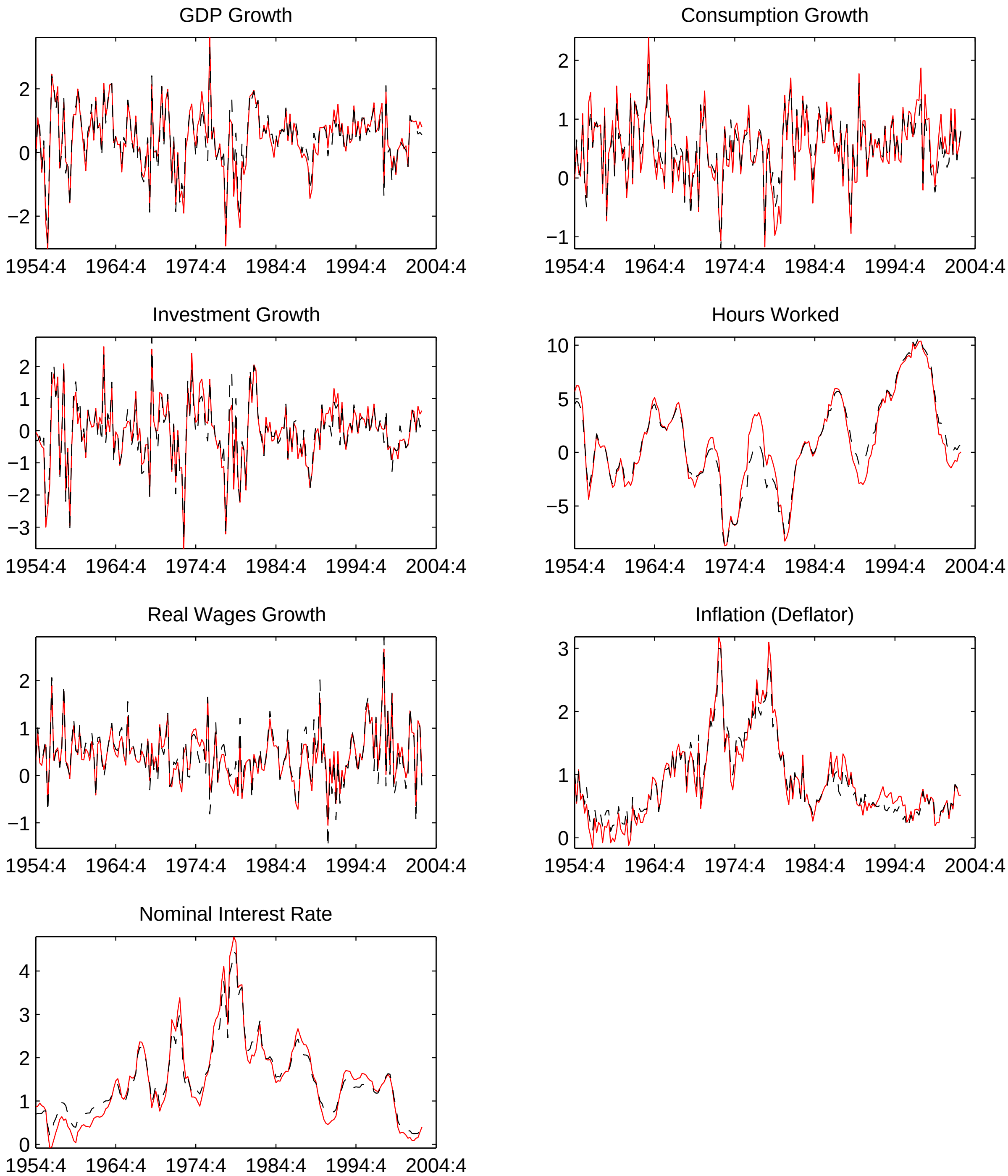


Figure 12. Fitted Observables: 'Short' Cov. Matrix Restrictions vs. Data



Note: estimate (red, solid), data (black,dashed)

VII. APPENDIX: DATA AND TRANSFORMATIONS

The data set and its transformation aim to follow the relevant literature discussed. Namely, for comparability reasons we follow the Appendix to Justiniano, Primiceri, and Tambalotti (2010). The data source is HAVER Analytics (USECON database) and FRED St. Louis database, series tickers in parenthesis. The balanced data range available is from 1954:Q3 to 2013:Q4. Real output is constructed using the value series (GDP), adjusted for the population (LF + LH) and the output deflator (JGDP). Real consumption and investment is also expressed in per capita terms, with the output deflator used to obtain the volume series. For better alignment with the model, the literature defines private consumption only as consumption of non-durable goods (CN) and services. Accordingly, private investment is defined as comprising gross private domestic investment (I) and consumption expenditures on durables (CD). Real wages are obtained as nominal compensation per hour in the non-farm business sector (LXNFC) adjusted for the output deflator. The effective Federal Funds rate (FFED) are taken as the model nominal rate of interest. The labor input is measured using hours of all persons in the nonfarm business sector (LXNFH), adjusted for populations and in logarithms.[11]

It should be understood that the data definition and transformations bear important consequences for the model analysis. First, the assumption of a single price in the model is at odds with a trend decline of relative price of investment goods to consumption goods, see e.g. . Second, the omission of both government consumption and net trade corrupts fundamental identities of national accounting in the model. Further, the scaling by population introduces a joint dynamics into consumption, investment, and output at particular frequencies. Further, the model has no mechanism for a potential shift in an implicit long-term inflation goal of the Fed and explains all frequencies of the U.S. inflation by demand and supply shocks.

[11] Justiniano, Primiceri, and Tambalotti (2010) report to use a series HNFBN from Haver Analytics. To our knowledge the series with this mnemonics is not present and has not ever been present in the Haver Analytics database. The representative of Haver Analytics were not able to identify the series or any discontinued series of that name and suggested the use of LXNFH series. I would like to thank Kevin Keithley (IMF) and Edlin Perdomo (Haver Analytics) for their help.

VIII. Appendix: Reduced-rank Filter Example

A. Simple Dynamic Semi-Structural Model

The model follows a typical New-Keynesian closed economy model with price rigidities. Inflation, π_t, is driven by output in excess of its trend or equilibrium value, using a forward-looking Phillips curve. The output cycle, $\hat{y}$, is determined by an output equation derived from consumption smoothing and is interest sensitive. The monetary policy authority sets the short-term nominal interest rate, i_t, via an inflation-forecast based rule, weighting the expected deviation of year-on-year inflation from its target and the output gap.

$$\hat{y}_t = \alpha_1 y_{t+1|t} + \alpha_2 y_{t-1} + \alpha_3 (rr_t - \overline{rr}_t) + \varepsilon_t^y \quad (11)$$

$$\pi_t^c = \lambda_1 \pi_{t+1|t}^c + (1-\lambda_1)\pi_{t-1}^c + \lambda_2 \hat{y}_t + \varepsilon_t^\pi \quad (12)$$

$$i_t = \gamma_1 i_{t-1} + (1-\gamma_1) \times \left[(\overline{rr}_t + \overline{\pi}_t) + \gamma_2 (\pi_{t+3|t}^{y/y} - \overline{\pi}_{t+3}) + \gamma_3 y_t \right] + \varepsilon_t^i \quad (13)$$

$$\pi_t = \pi_t^c + \varepsilon_t^{\pi,sr} \quad (14)$$

$$rr_t = i_t - \pi_{t+1|t} \quad (15)$$

$$\overline{\pi}_t = \rho_{\overline{\pi}} \overline{\pi}_{t-1} + \varepsilon_t^{\overline{\pi}} \quad (16)$$

Despite its small size and simplicity, the model can display nontrivial dynamics in response to structural shocks. It is driven by eight parameters $\theta = \{\alpha_{1,2,3}, \lambda_{1,2}, \rho_i, \gamma_{1,2,3}\}$ and four standard deviations for structural shocks. Inflation target is assumed to follow a random-walk, $\rho_{\overline{\pi}} = 1$.

B. State Estimation with the Model

State estimation with the model can proceed using the Kalman smoother, as it is standard. To illustrate the SVD filter, it is assumed that due to misspecification of the model, the researcher attempts to identify only the effect of the demand shocks, ε_t^y.

To make the state-estimation test interesting, the model is simulated with non-zero variance of the demand, ε_t^y, both inflation shocks (long, ε_t^π, and short, $\varepsilon_t^{\pi,sr}$), as well as with the policy shock, ε_t^i. The standard deviations and other model coefficients are stated in the appendix. Obviously, the smaller is the signal of the particular shock to recover, the harder it will be. In addition, the random simulation of the model's structural shocks was augmented with identically independently distributed random 'measurement errors', which add high-frequency variation to the actual observed data.

Given the nature of the SVD filter, if only the iid measurement noise to all variables was added, without the structural shocks being sampled, the demand shock is identified perfectly.[12] The reason is that the impulse-response function of the demand shock can in no way replicate such erratic dynamics and thus extract the true shock with high precision, completely ignoring the high-frequency band that the measurement shocks occupy. For this no changes in the model's setup are needed, that is automatic within the SVD filter.

As can be viewed from Fig. 13, in the presence of both the measurement noise and other structural shocks than demand the identification of the demand shock is not perfect. It is not, however, hopelessly identified. It easily sees through the noisy variations but–as expected– the identified shock is a linear combination of the structural ones. Not all is lost, however, as can be viewed from the Fig. 14, where the comparison of the dynamics solely due to the true demand shock and the dynamics due to estimated shocks are contrasted. The other shocks, namely the long inflation shock,ε_t^{π}, confound the information in a nontrivial way and the estimated dynamics differ from the true ones.

[12] Results upon request for this exercise.

Figure 13. Noisy data, structural shocks, and estimates

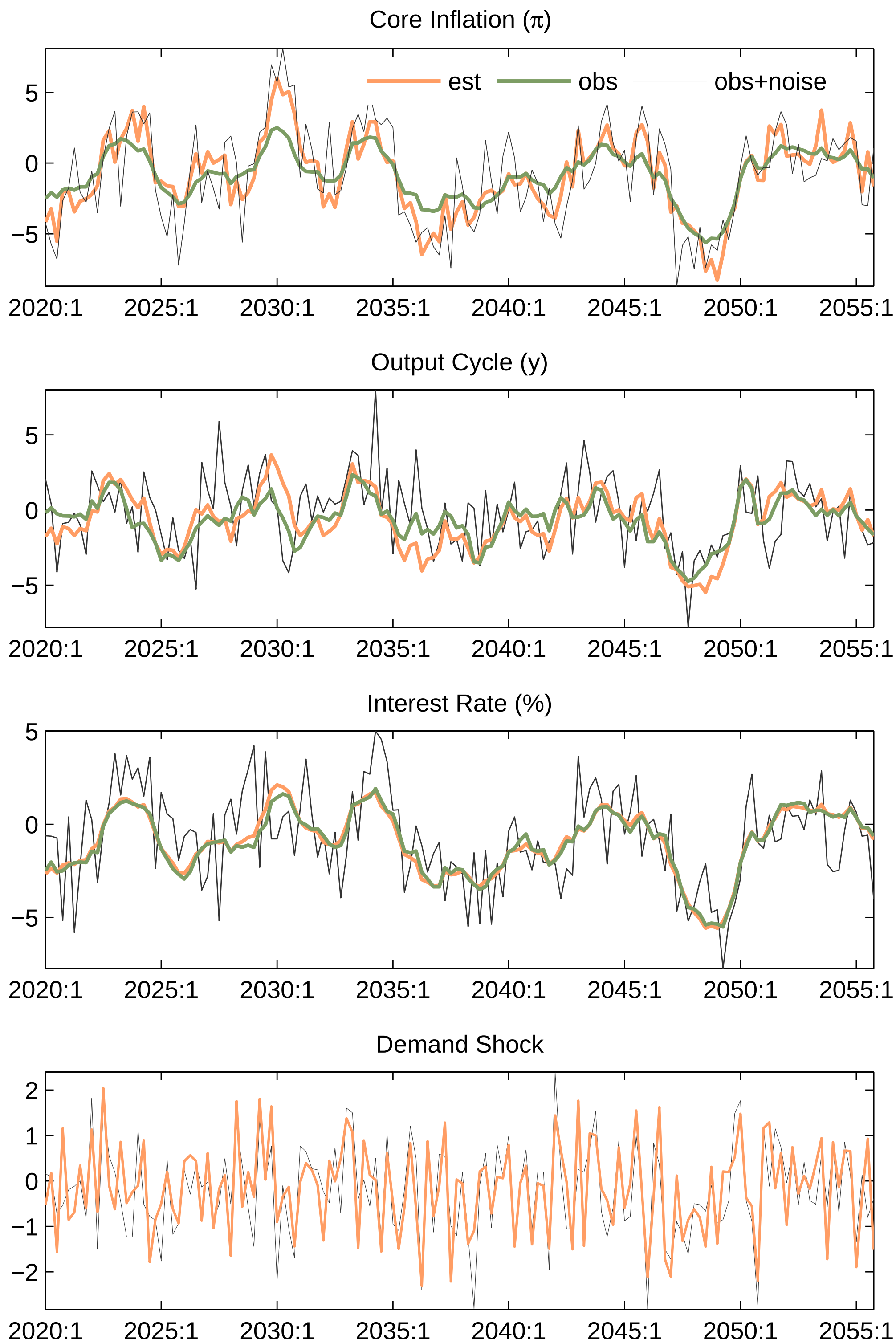


Figure 14. Effects of true vs. estimated demand shock