Estimating Structural Shocks with DSGE Models

Michal Andrle

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Outline of the Talk

- **‘Structural’ shocks?**
- **Model vs. Filter Misspecification**
  - Propagation mechanisms, economics
  - Number of shocks, residuals
- **Bliss of Stochastic Singularity**
  - Factor analysis of the OECD data
  - Give me my residuals baaack!
  - SVD filtering
- **Evil of Correlated shocks**
  - Causes & consequences
  - Testing for misspecification
  - PROCRUSTES filtering
- **Illustrations. . .**

'Structural' Shocks…

WE NEED STRUCTURAL SHOCKS
TO TELL ECONOMICALLY MEANINGFUL STORIES…

Operational definition of 'structural shock':

1. You can tell reasonable, economically meaningful stories
2. Shocks are not systematically cross-correlated

Misspecification:

DSGE models **start** with uncorrelated (independent) shocks that render IRFs, FEVDs, etc. meaningful…

…but end up with correlated shocks in most cases.

Issues & Suggested Solutions:

1. **Model vs. Filter Misspecification**
   a) model's economics can be off
   b) economics is right but the filter is ill-conditioned
2. **How many structural shocks can we hope to identify?**
   - factor/DPCA analysis of OECD data suggest just few key shocks
   - a few shocks responsible for bulk of dynamics, bulk of shocks contributing little to data dynamics
   - is 'stochastic singularity' actually a blessing?

Analysis in the paper:

1. **Testing for misspecification:** correlated shocks
2. **SVD Filtering:** dealing with stochastic singularity
3. **Procrustes Filtering:**

   (can we explaining data with uncorrelated shocks?)

Misspecification Sign: Correlated Shocks

Assume a linear model with structural shocks, ε,

$$Y_t = D(L)\varepsilon_t \quad \varepsilon_t \sim N(0, I) \tag{1}$$

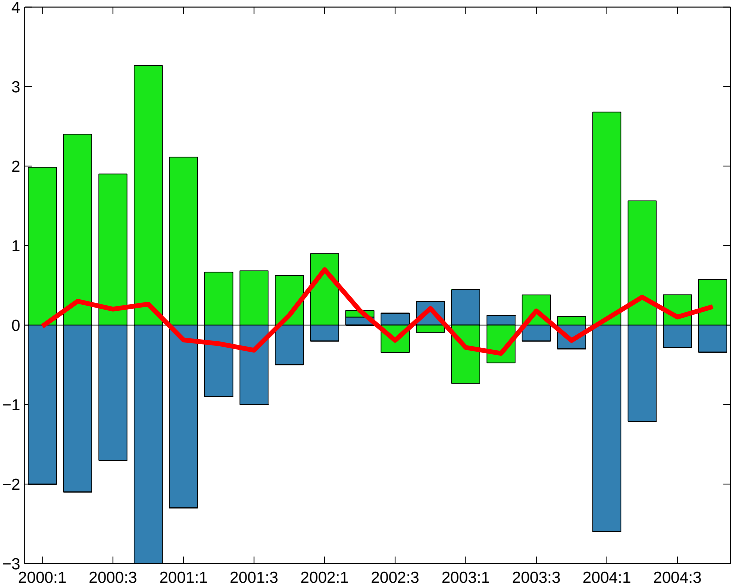
and associated filter $F(L)$ for uncovering structural shocks,

$$\hat{\varepsilon}_t = F(L)Y_t. \tag{2}$$

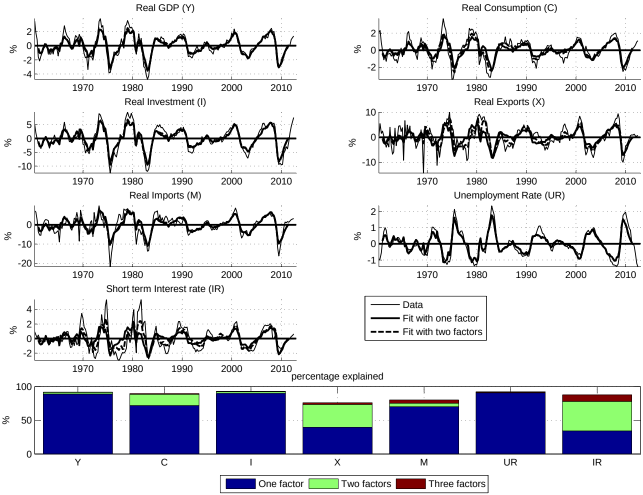
Sufficient (but not necessary) condition for **misspecification** is when estimated shocks, $\hat{\varepsilon}_t$ are **significantly** cross- and auto-**correlated**.

Leads to "fishy" shock decompositions...

Cross-Correlated Shocks: Fishy FISH



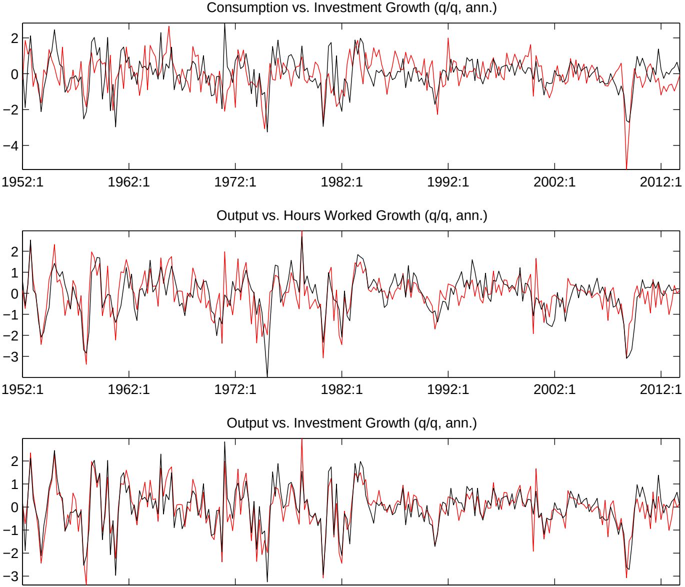
Example: Stochastic Rank of US Data since 1950



Andrle, Brůha, Solmaz (2014)

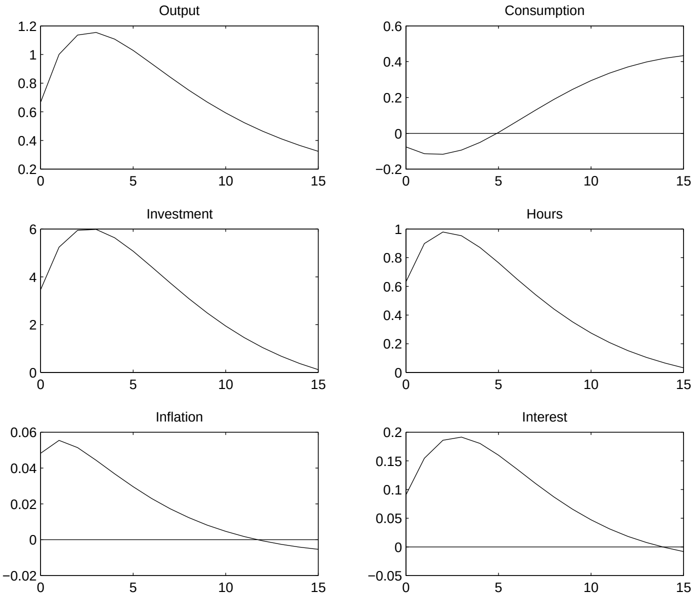
Example: Stochastic Rank of US Data (JPT 2010)

Justiniano, Primiceri, and Tambalotti (JME, 2010) (JPT 2010)



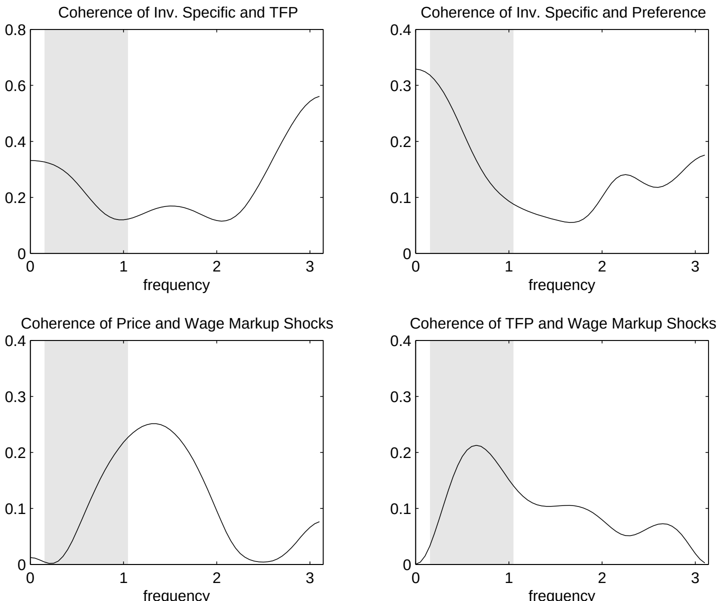
Typical Investment Shock (JPT 2010)

. . . but 'risk' shocks and credit shocks on the same boat, essentially



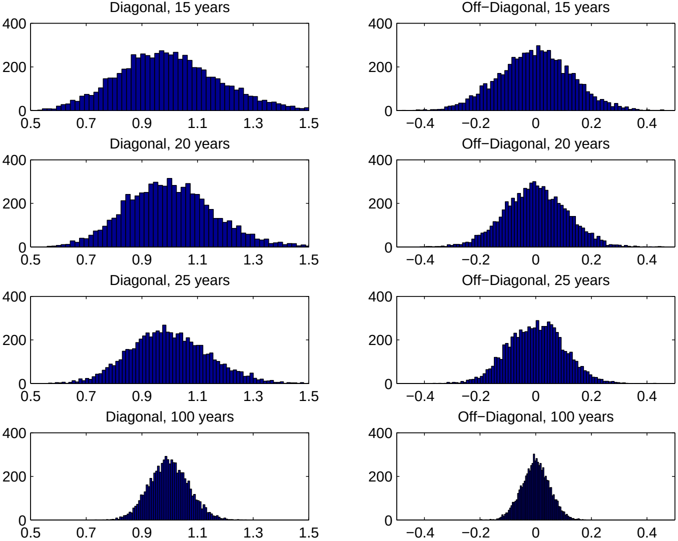
Shock Cross-Correlation & Misspecification

... not a white noise, really



Misspecification Sign: Correlated Shocks

JPT-2010: correlation of inv. shock(t) and pref. shock(t-1) is 0.28



Finite sample distributions of $N(0, I_k)$

Ill-Conditioning and Fragility

Even with a great model, **Kalman smoothing** is a rather **fragile** exercise that does **not** aspire for **robustness.**

Shock identification is a **signal-extraction exercise**, Kalman filter must be viewed as as a filter: $\varepsilon_t = F(L) Y_t$ (Andrle 2013a, 2013b)

The filter $F(L)$ may be very **ill-conditioned**, i.e. over-sensitive to small changes in the input data, Y. It's a property of the model/filter!

The goal of explaining **100% of data dynamics** with 'structural shocks' and a stylized model is fraught with **hazards**

What's wrong with having residuals? Your OLS has it...

Robustness: Roughly Right or Precisely Wrong?

Ill-conditioning and fragility:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1+\phi & 1 \\ 1 & 1+\theta \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \tag{3}$$

for $\theta \to 0$ the model is over-sensitive to changes in y.

$$\begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} -0.1 \\ -0.3 \end{bmatrix} \xrightarrow{A} \begin{bmatrix} -0.4 \\ -0.4 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \qquad \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -0.4000 \\ -0.3999 \end{bmatrix} \xrightarrow{A} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} = \begin{bmatrix} -0.6 \\ 0.2 \end{bmatrix}$$

Even with the right model, a small perturbation to input data leads to important changes in the estimates of structural shocks...

Below it is demonstrated that Kalman smoother has the form $\mathbf{Y} = \mathbf{A} \times \mathbf{E}$ as above

State-Space Setup

The linear state-space model is given by:

$$Y_t = ZX_t + H\varepsilon_t \quad (4)$$
$$X_t = TX_{t-1} + R\varepsilon_t \quad \text{with} \quad \varepsilon_t \sim N(0, I) \quad (5)$$

The model is said to be **stochastically singular** if

$$\text{rank}\, S_y(\omega) < \text{rank}\, S_\varepsilon(\omega).$$

For singularity it is sufficient that $n_\varepsilon < n_Y$.

Today, we'll stay in **time domain** but **frequency domain** would serve just fine as well…

Kalman Smoother as a Least-Squares Problem (1)

$$\min_{X_0,\{\varepsilon\}} \quad \Lambda = X_0 P^{-1} X_0 \quad + \quad \sum_{t=1}^{N} [Y_t - ZX_t]\,(HH')^{-1}\,[Y_t - ZX_t]' \\ + \quad \sum_{t=1}^{N} [X_t - TX_{t-1}]\,(RR')^{-1}\,[X_t - TX_{t-1}]' \,.$$

The genius of Kalman was to make the problem recursive!

. . . and I'm kind of doing the opposite

Kalman Smoother as a Least-Squares Problem (2)

Denoting $\mathbf{Y} = [Y_1 \;\; Y_2 \;\; \dots \;\; Y_N]'$, $\mathbf{E} = [\varepsilon_1 \; \varepsilon_2 \;\; \dots \;\; \varepsilon_N]$, and $\mathbf{Z} = [X_0 \; \mathbf{E}]$, the least-squares problem is stated as follows:

$$\mathbf{Z} = \text{argmin} \, ||\text{vec}\mathbf{Y} - \mathbf{A} \times \text{vec}\, \mathbf{Z}||, \tag{6}$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{ZT} & | & \mathbf{ZR}+\mathbf{H} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{ZT}^2 & | & \mathbf{ZT}^2\mathbf{R} & \mathbf{ZTR} & \mathbf{ZR}+\mathbf{H} & \mathbf{0} & \dots & \mathbf{0} \\ \vdots & | & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathbf{ZT}^N & | & \mathbf{ZT}^{N-1}\mathbf{R} & \mathbf{ZT}^{N-2}\mathbf{R} & \dots & \dots & \dots & \mathbf{ZR}+\mathbf{H} \end{bmatrix} = [\mathcal{O} \; \mathcal{H}]. \tag{7}$$

SVD Filtering (1): Handles Singular Models

"SVD Filter" – shock estimates for both regular and singular problems

$$\mathbf{A} = \begin{bmatrix} \mathbf{U}_1 & \mathbf{U}_2 \end{bmatrix} \begin{bmatrix} \mathbf{S}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{V}_1' \\ \mathbf{V}_2' \end{bmatrix} = \sum_{i=1}^{r} s_i u_i v_i'. \tag{8}$$

where $r = \text{rank}(\mathbf{A})$, $\mathbf{U}'\mathbf{U} = \mathbf{I}$, $\mathbf{V}'\mathbf{V} = \mathbf{I}$ and $\mathbf{S}_1$ is diagonal, $\mathbf{S}_1 = \text{diag}(s_1, \ldots, s_p)$, where $p = \min\{m, n\}$. When the matrix $\mathbf{A}$ is singular, then rank of $\mathbf{A}$ is r, then $s_{r+1} = \cdots = s_p = 0$.

The solution to $\text{vec}\,\mathbf{Z}$ is then obtained as

$$\begin{aligned} \text{vec}\,\mathbf{Z} &= \mathbf{V}_1 \mathbf{S}_1^{-1} \mathbf{U}'_1 \times \text{vec}\,\mathbf{Y} && (9) \\ &= \sum_{i=1}^{r} \frac{u_i' \times \text{vec}\,\mathbf{Y}}{s_i} v_i. && (10) \end{aligned}$$

SVD Filtering (2)

SVD Filter is an extension of the Kalman filter:

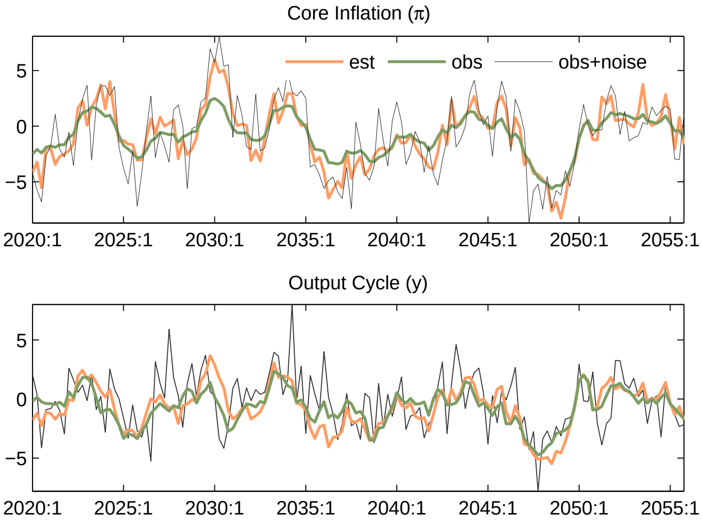
(i) For regular models is identical to Kalman smoother/filter

(ii) Handles singular models

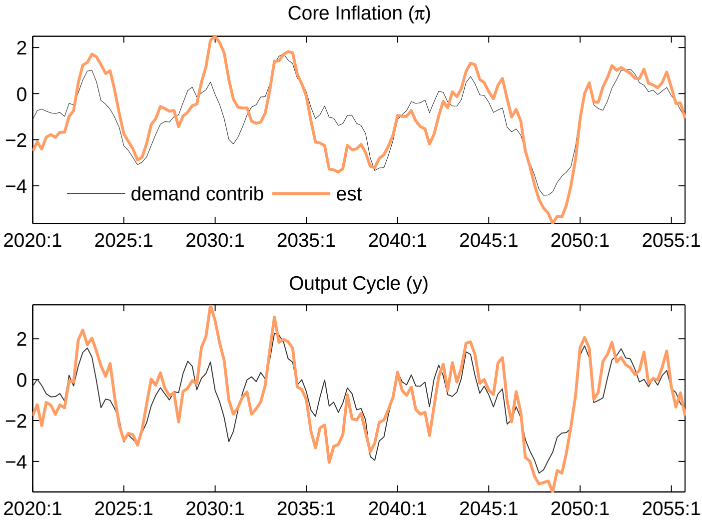
(iii) Makes inspecting of ill-conditioning a cinch (SVD of **A**)

(iv) Implementation in frequency domain is more efficient (memory, speed)

SVD Filter Example (1)



SVD Filter Example (2)



Procrustes Filtering

- We minimize distance of the model and data in a mean-square sense (Kalman smoother)
- Using SVD filter we can choose any number of shocks we want to test and distinguish residuals and structural shocks
- So how about choosing the shocks only from a set of ‘reasonably uncorrelated’ shocks?

Question asked:

How big residuals I get when I fit the data with shocks that can’t be too correlated? If I cannot fit much, am I in BIG trouble?

Intuition – Orthogonal Procrustes Problem

Given **A** and **B**, find orthogonal rotation matrix **R** such that

$$\mathbf{R} = \text{argmin}||\mathbf{R} \times \mathbf{A} - \mathbf{B}||_F \;\; \text{s.t.} \;\; \mathbf{R}'\mathbf{R} = \mathbf{I} \tag{11}$$

There is an **analytical** solution to orthogonal Procrustes...

“Procrustes Filter”: Uncorrelated Shocks Imposed

Purpose:
Given the model, estimate structural shocks to match the data dynamics as best as possible, while keeping the shocks uncorrelated…

$$\mathbf{Z}_\lambda = \text{argmin}\,\{||\text{vec}\mathbf{Y} - \mathbf{A} \times \text{vec}\,\mathbf{Z}|| + \lambda||N^{-1}\mathbf{E} \times \mathbf{E}' - \mathbf{I}||_{\mathcal{M}}\},$$

Here, $||.||_{\mathcal{M}}$, is a suitable matrix measure

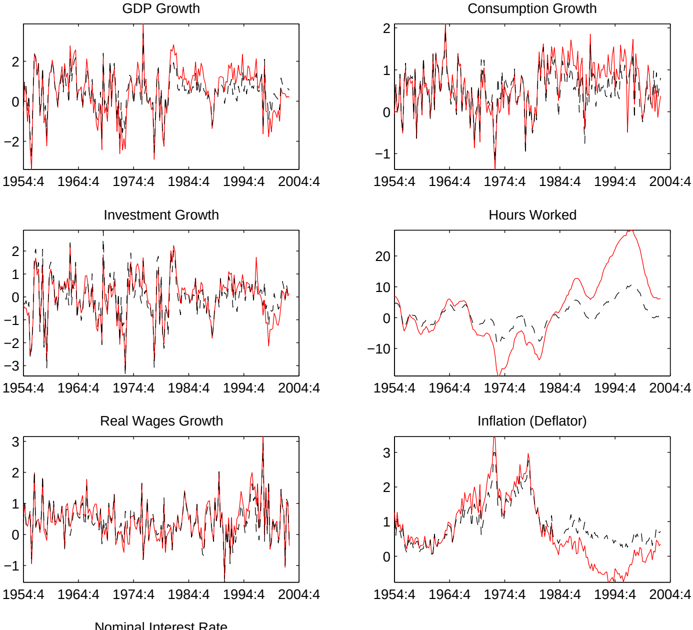
- L2 distance
- penalty function reflecting finite-sample variation

Acknowledging the finite-sample distribution of shocks is important.

“Procrustes Filter”: Weak & Strong Version

1. **Weak PF**: penalizes contemporaneous cross-cov ONLY
2. **Strong PF**: penalizes the ACGF/spectrum profile

Example: ‘Weak’ Procrustes using JPT-2010



Example: ‘Strong’ Procrustes using JPT-2010

Thank you for your patience. . .