DSGE2VAR:
Mapping Structural Models to VAR Models

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I. Introduction

This short note proposes a **simple** and **practical** mapping between VAR and structural models, DSGE models for instance. The use of the mapping is to investigate the differences in strucutral interpretation of the data with the two type of models. If two models are used to explain an identical set of data, there exist an implicit mapping betwen identified structural shocks in both models.

The premise—and the only precondition–is that the data set and data transformations used in both models are identical. More precisely, that the VAR data set is a proper subset of the dataset used in the structural economic model.

Unlike **?** or **?** this is not an attempt to demonstrate under what conditions a particular DSGE model has a finite or infinite VAR representation. The goal is simply to see clearly the implication of a structural model for a reduced-form (VAR) model with structural identification (SVAR) consistent with the structural model or with identical data. This is useful for DSGE-SVARs in particular.

II. The Mapping

Assume the observed data is a $(n_y \times 1)$ vector Y_t, which is a subset of the observables in both models. Further, assume that both models can be expressed in a moving average form.

The DSGE model is specified as

$$Y_t = D(L|\theta)\eta_t, \tag{1}$$

where η_t is a $(m \times 1)$ vector of identified structural shocks, with $m \geq n$. The notation explicitly acknowledges the fact that the MA coefficients are a function of structural parameters θ. For simplicity, the case of stochastic singularity is not considered—it would have grave implications for the VAR model.

The SVAR model–with all shocks identified–is specified using a regular moving average representation as

$$Y_t = C(L|\Psi)\varepsilon_t, \tag{2}$$

where ε_t is a $(n \times 1)$ vector of identified structural shocks. By nature of identified VAR, the polynomial $C(L)$ is easily invertible and squared.

Since both models are purported to explain the identical subset of variables, then

$$C(L)\varepsilon_t = Y_t = D(L)\eta_t \tag{3}$$

and thus

$$\varepsilon_t = C(L)^{-1}D(L)\eta_t = F(L)\eta. \tag{4}$$

It is obvious that structural shocks in a SVAR are a moving average of the DSGE structural shocks.

III. Implementation

The implementation is very easy. For finite samples, though, one must take care of initial conditions in both models.

With no loss of generality, assume the SVAR is of lag $p = 1$, i.e. $Y_t = AY_{t-1} + R\varepsilon_t$. Assume a standard state-space representation of the structural DSGE model as

$$\begin{aligned} Y_t &= ZX_t + K\eta_t \qquad (5) \\ X_t &= TX_{t-1} + H\eta_t. \qquad (6) \end{aligned}$$

For Y_0 and $X_{0,-1}$ are given. The first period structural shocks map to each other as follows:

$$\varepsilon_1 = R^{-1}[ZTX_0 - AY_0] + [ZH + K]\eta_1, \tag{7}$$

since both model must yield identical value of Y_1 given the initial conditions and unexpected structural shocks. The recursion proceeds with

$$R\varepsilon_2 = [ZT^2X_0 - A^2Y_0] - AR\varepsilon_1 + ZTH\eta_1 + [ZH + K]\eta_2, \tag{8}$$

into which ((7)) can be easily plugged to eliminate ε_1. This is redundant for the actual computation of course but useful to have a closed-form for $F(L)$.

IV. CONCLUSION

The mapping from a DSGE model to VARs is very simple to investigate. This little testing procedure can be useful for development of DSGE-VAR models or for supplemental testing for IRF matching estimators of DSGE models.