Exact Non-Linear Smoother for DSGE Models

driver_test.m

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Introduction

This tutorial demonstrates how to build your own tools building on the flexibility and stable interface of IRIS. I am hugely indebted to Jaromir, without him I wouldn't work on this at all...

An exact non-linear filter (smoother) for non-linear DSGE models is based on a simple idea of using non-linear least squares on top of repeated solution of the non-linear model. Kalman filter is a linear least squares, hence below you find a simple tool for non-linear least squares for IRIS models.

The code of the filter `nlfilter.m` is suited mostly for smaller models with modest non-linearity, not for estimation. The limitation of this simple, truly a brute-force method is time and IRIS capability to solve the model every step of the way. Also, due to the solution method a Jensen inequality is ignored, a price paied for retainig a non-linearity.

The tutorial clearly examplifies the importance and utility of IRIS' object oriented design with a well documented and stable public interface to its capabilities. The logic of the filter, however, will work with any other software able to solve a non-linear DSGE model.

Contents

1 Principles of the Exact Nonlinear Smoother

The non-linear exact filter is based on a simple idea that if a linear Kalman filter is equivalent to linear least squares, a non-linear exact Kalman filter should can be based on non-linear least squares.

This is simple and well understood in the literature. For purely backward-looking, recursive models this principle is exploited and used in the engineering literature. However, due to rational expectations the models in economics are more complex.

The exact non-linear least squares filter is not feasible for estimation of the model, it would just take too much time. It is mainly for understanding of the shocks behind the non-linear model, identifed from the data if you have few minutes to spare.

Remember what we are doing here: we are solving a non-linear least squares problem on top of a model solution, searching for a path of shocks and set of initial conditions, that explain the observed data and are most likely given the likelihood of the shocks. The more periods and the more shocks, the larger is the problem.

Let's dive into the example...

2 House keeping

```
clear all; close all; clear classes; clc
```

3 Loading the Endocred model

```
read_endocred;
```

```
                                        First-Order                    Norm of
 Iteration  Func-count     Residual      optimality      Lambda           step
     0           7          4.21225            2.24        0.01
     1          14         0.225278           0.249       0.001        2.76598
     2          21       0.00264272            0.02      0.0001        2.70455
     3          28      0.000341442          0.0111      1e-005       0.564446
     4          35     9.56255e-011        5.78e-006     1e-006      0.0329728
     5          42     1.33257e-019        3.96e-011     1e-007   2.95614e-005
     6          49     5.54841e-029        8.88e-016     1e-008   5.19942e-009
     7          56     1.97215e-033        4.44e-018     1e-009   1.06481e-013

Local minimum found.

Optimization completed because the size of the gradient is less than
the selected value of the function tolerance.
```

```
                                         First-Order                      Norm of
 Iteration  Func-count     Residual       optimality      Lambda           step
     0           3               5                2         0.01
     1           6     0.000490148           0.0198        0.001        2.21393
     2           9    4.89171e-010        1.98e-005       0.0001      0.0221172
     3          12    4.89056e-018        1.98e-009       1e-005    2.2115e-005
     4          15    4.97173e-028           2e-014       1e-006   2.21144e-009
     5          18    9.78121e-041        9.89e-021       1e-007   2.22974e-014

Local minimum found.

Optimization completed because the size of the gradient is less than
the selected value of the function tolerance.



ans =
        y_obs: 0
       pi_obs: 3.0000
            y: 0
           pi: 3.0000
          pi4: 3.0000
            r: 5.0000
            t: 3
            c: 1
        eps_y: 0
       eps_pi: 0
           ey: 0
          epi: 0
           et: 0
           er: 0
          alp: 0.5000
          sgm: 0.1000
          bet: 0.9900
          gam: 0.0900
          del: 0.4000
          the: 0.8000
          kap: 4
          phi: 0
          rho: 2
          tau: 3
          psi: 0.4000
          omg: 1.3000
```

```
     std_eps_y: 0.0100
    std_eps_pi: 0.0100
        std_ey: 0.0100
       std_epi: 0.0100
        std_et: 0.0100
        std_er: 0.0100
        non-linear model object: 1 parameterisation(s)
        solution(s) available for a total of 1 parameterisation(s)
        comment: 'Simple endogenous credibility model'
        user data: empty
```

4 Setting up the range and the model parameters

```
rng = qq(2000,1):qq(2012,4);

m1 = m;
m1.std_eps_y = 0.00001;
m1.std_eps_pi = 0.00001;
m1.std_ey = 0.03;
m1.std_epi = 0.07;
```

5 Get a random path of shocks to simulate a test case

I resample the model in a linear way as a simple way of getting the shocks and paths of variables. Yet, as non-linear simulation as a base case `truth` is needed I re-simulate the model with nonlinearities. Allowing the `display` option visible in this case shows you the progress of the quasi Newton iteration of IRIS while solving a model...

```
%d = sstatedb(m1,rng,'randomshocks=',true);
d = resample(m1,[],rng,1);

ds = simulate(m1,d,rng,'anticipate', false,...
                  'nonlinearise', length(rng),...
                  'maxiter', 5000,'display',false);

% clear screen
clc;
```

6 Comparing the Linear filter to Non-linear prediction error filter

linear filter

```
[ans smth] = filter(m1,ds,rng);
dbf2 = smth.mean;

% filter with non-linear pred. error
[ans smth] = filter(m1,ds,rng,'nonlinearize',10);
dbf3 = smth.mean;
```

```
      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      1:1[10]      0 1.87184e-015 Phillips curve           1            0 Phillips curve
      1:1[10] =====0 1.87184e-015 Phillips curve           1            0 Phillips curve

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      2:2[11]      0   0.00269157 Credibility              1            0 Phillips curve
      2:2[11] =====2 8.42847e-006 Phillips curve           1   0.00261218 Credibility

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      3:3[12]      0   0.00186281 Credibility              1            0 Phillips curve
      3:3[12] =====2 8.92074e-006 Phillips curve           1   0.00181455 Credibility

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      4:4[13]      0   0.00167735 Credibility              1            0 Phillips curve
      4:4[13] =====2  5.8702e-006 Credibility              1   0.00161273 Credibility

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      5:5[14]      0   0.00268683 Credibility              1            0 Phillips curve
      5:5[14] =====2 8.08882e-006 Phillips curve           1   0.00262073 Credibility

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      6:6[15]      0   0.00161655 Credibility              1            0 Phillips curve
      6:6[15] =====2 5.57358e-006 Phillips curve           1   0.00159167 Credibility

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      7:7[16]      0     0.001685 Credibility              1            0 Phillips curve
      7:7[16] =====2 5.72633e-006 Phillips curve           1   0.00165702 Credibility

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      8:8[17]      0  0.000230951 Credibility              1            0 Phillips curve
      8:8[17] =====1 8.62729e-006 Phillips curve           1  0.000230951 Credibility

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
      9:9[18]      0   0.00146065 Credibility              1            0 Phillips curve
      9:9[18] =====2 9.87731e-006 Phillips curve           1   0.00146065 Credibility

      Segment   Iter  Max.discrep Equation            Lambda  Max.addfact Equation
    10:10[19]      0   0.00150364 Credibility              1            0 Phillips curve
```

```
  10:10[19] =====2 6.28807e-006 Credibility               1   0.00150364 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  11:11[20]      0   0.00260678 Credibility               1            0 Phillips curve
  11:11[20] =====2 5.08932e-006 Phillips curve            1   0.00260678 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  12:12[21]      0  0.000241508 Credibility               1            0 Phillips curve
  12:12[21] =====1  7.6981e-006 Phillips curve            1  0.000241508 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  13:13[22]      0   0.00608113 Credibility               1            0 Phillips curve
  13:13[22] =====3 5.63771e-006 Phillips curve            1   0.00612429 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  14:14[23]      0    0.0026546 Credibility               1            0 Phillips curve
  14:14[23] =====2 8.85672e-006 Credibility               1    0.0026962 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  15:15[24]      0   0.00104654 Credibility               1            0 Phillips curve
  15:15[24] =====2  9.5341e-006 Phillips curve            1   0.00107014 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  16:16[25]      0   0.00153735 Credibility               1            0 Phillips curve
  16:16[25] =====2 8.72698e-006 Phillips curve            1   0.00152358 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  17:17[26]      0   0.00330272 Credibility               1            0 Phillips curve
  17:17[26] =====2 9.90466e-006 Credibility               1   0.00327113 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  18:18[27]      0   0.00441681 Credibility               1            0 Phillips curve
  18:18[27] =====3 9.88914e-006 Phillips curve            1   0.00437737 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  19:19[28]      0   0.00507112 Credibility               1            0 Phillips curve
  19:19[28] =====3  9.8092e-006 Phillips curve            1   0.00508951 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  20:20[29]      0   0.00344497 Credibility               1            0 Phillips curve
  20:20[29] =====3 8.55576e-006 Credibility               1   0.00345994 Credibility

    Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
  21:21[30]      0   0.00911321 Credibility               1            0 Phillips curve
  21:21[30] =====3 7.75547e-006 Phillips curve            1   0.00916363 Credibility
```

```
  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
22:22[31]      0   0.00733824 Credibility                1            0 Phillips curve
22:22[31] =====3 6.71492e-006 Credibility                1   0.00733905 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
23:23[32]      0   0.00188411 Credibility                1            0 Phillips curve
23:23[32] =====3  7.5767e-006 Credibility                1   0.00191069 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
24:24[33]      0   0.00279973 Credibility                1            0 Phillips curve
24:24[33] =====2 8.92224e-006 Phillips curve             1   0.00279973 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
25:25[34]      0   0.00211627 Credibility                1            0 Phillips curve
25:25[34] =====2 9.60783e-006 Phillips curve             1   0.00211627 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
26:26[35]      0   0.00258847 Credibility                1            0 Phillips curve
26:26[35] =====3 9.92672e-006 Credibility                1   0.00260013 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
27:27[36]      0    0.0139079 Credibility                1            0 Phillips curve
27:27[36] =====4 9.08832e-006 Phillips curve             1    0.0141505 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
28:28[37]      0    0.0114784 Credibility                1            0 Phillips curve
28:28[37] =====4 9.65099e-006 Phillips curve             1    0.0117333 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
29:29[38]      0    0.0146419 Credibility                1            0 Phillips curve
29:29[38] =====5 9.09349e-006 Credibility                1    0.0152373 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
30:30[39]      0    0.0032927 Credibility                1            0 Phillips curve
30:30[39] =====2 9.27925e-006 Credibility                1   0.00323493 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
31:31[40]      0   0.00278437 Credibility                1            0 Phillips curve
31:31[40] =====2 7.64603e-006 Phillips curve             1   0.00274212 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
32:32[41]      0   0.00359245 Credibility                1            0 Phillips curve
32:32[41] =====2 8.84591e-006 Credibility                1   0.00345175 Credibility

  Segment   Iter  Max.discrep Equation              Lambda  Max.addfact Equation
33:33[42]      0    0.0100396 Credibility                1            0 Phillips curve
```

```
33:33[42] =====3 9.14843e-006 Credibility              1   0.00997997 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
34:34[43]      0   0.00236833 Credibility              1            0 Phillips curve
34:34[43] =====2 9.90667e-006 Phillips curve           1   0.00231963 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
35:35[44]      0   0.00564357 Credibility              1            0 Phillips curve
35:35[44] =====3 8.46678e-006 Phillips curve           1   0.00565875 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
36:36[45]      0   0.00669181 Credibility              1            0 Phillips curve
36:36[45] =====3 9.12484e-006 Phillips curve           1    0.0067397 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
37:37[46]      0  0.000693346 Credibility              1            0 Phillips curve
37:37[46] =====1 9.45641e-006 Phillips curve           1  0.000693346 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
38:38[47]      0   0.00668727 Credibility              1            0 Phillips curve
38:38[47] =====3 9.23538e-006 Phillips curve           1   0.00670085 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
39:39[48]      0   0.00184637 Credibility              1            0 Phillips curve
39:39[48] =====2 9.09246e-006 Phillips curve           1   0.00182158 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
40:40[49]      0   0.00175625 Credibility              1            0 Phillips curve
40:40[49] =====2  9.8928e-006 Phillips curve           1   0.00171933 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
41:41[50]      0   0.00188185 Credibility              1            0 Phillips curve
41:41[50] =====3  9.5369e-006 Phillips curve           1   0.00184221 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
42:42[51]      0  0.000987422 Credibility              1            0 Phillips curve
42:42[51] =====2 7.57802e-006 Phillips curve           1     0.001004 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
43:43[52]      0   0.00171651 Credibility              1            0 Phillips curve
43:43[52] =====2 9.57287e-006 Phillips curve           1   0.00166418 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
44:44[53]      0   0.00224617 Credibility              1            0 Phillips curve
44:44[53] =====2  9.2645e-006 Credibility              1    0.0023162 Credibility
```

```
  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
45:45[54]      0   0.00286813 Credibility               1            0 Phillips curve
45:45[54] =====3 7.71281e-006 Credibility               1   0.00296993 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
46:46[55]      0   0.00308697 Credibility               1            0 Phillips curve
46:46[55] =====2 9.55249e-006 Credibility               1   0.00326448 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
47:47[56]      0   0.00254399 Credibility               1            0 Phillips curve
47:47[56] =====3 7.93321e-006 Phillips curve            1   0.00262969 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
48:48[57]      0   0.00913676 Credibility               1            0 Phillips curve
48:48[57] =====3 9.50164e-006 Credibility               1   0.00927913 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
49:49[58]      0   0.00172153 Credibility               1            0 Phillips curve
49:49[58] =====2 8.87544e-006 Phillips curve            1   0.00173701 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
50:50[59]      0    0.0007082 Credibility               1            0 Phillips curve
50:50[59] =====1 8.74244e-006 Phillips curve            1    0.0007082 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
51:51[60]      0   0.00415974 Credibility               1            0 Phillips curve
51:51[60] =====3 9.67918e-006 Credibility               1   0.00413736 Credibility

  Segment   Iter  Max.discrep Equation             Lambda  Max.addfact Equation
52:52[61]      0   0.00238654 Credibility               1            0 Phillips curve
52:52[61] =====2 7.85301e-006 Phillips curve            1    0.0023621 Credibility
```

7 Exact non-linear smoother (brute force least squares)

Now, let's compare also the non-linear least square approach to the linear filter and to the non-linear filter in IRIS, based on the non-linear prediction error.

First, an instance of the class `nlfilter.m` needs to be initalized. Second, a solved IRIS model is passed to the filter, `initIrisModel()`. The user have full control on the way how the model is repeatedly simulated, i.e. maximum numbe of iteration, if shocks are surprises or if it is solved in a non-linear way at all. The standard model options are passed to `setSimulOpts` to initialize the filter. In an analogous way, the user can set the options for the least squares solver, see the help.

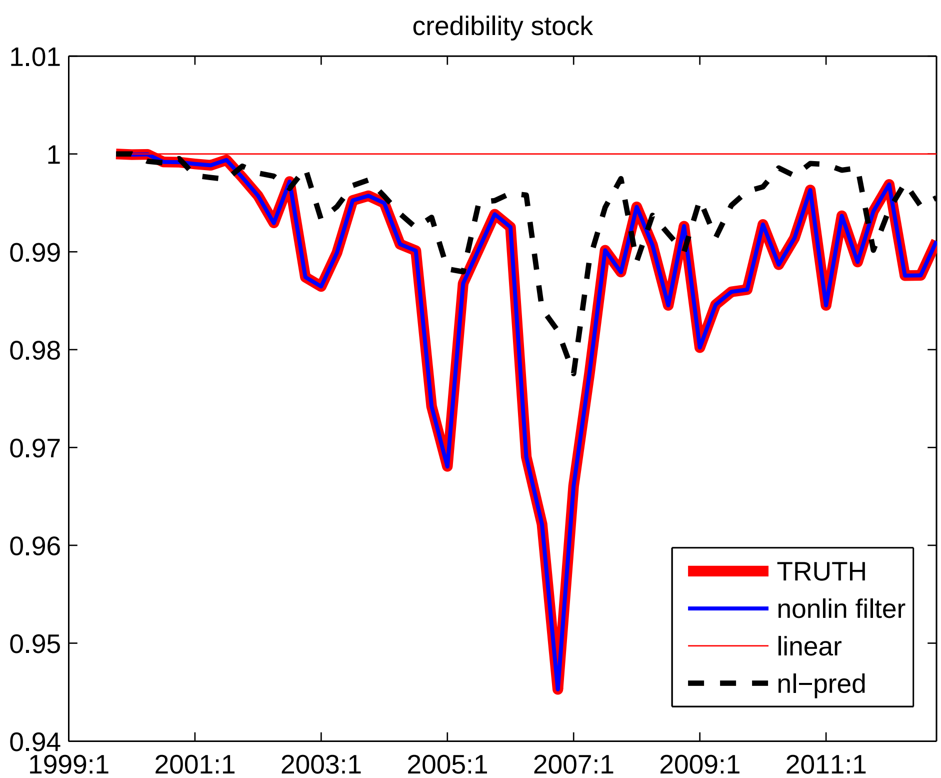
Having the initialized filter, the filter is run using the `filter` method with the input being the IRIS database and range. Starting values are based on the steady-state of the model, though using `startDb` method you can provide your own, potentially...

NOTE: This example will run for you if and only if you have access to Matlab optimizing Tbx. For help and other options type `help/nlfilter` or investigate the code.

```
n = nlfilter();

  % initialize the |nlfilter|
  n.initIrisModel(m1);

  % option for a model simulation
  n.setSimulOpts('anticipate',false,...
                'maxiter', 3000,...
                'nonlinearize', length(rng),...
                'display',false);

  % run the nl-filter
  df1 = n.filter(ds,rng);


% credibility stock
f = figure();
        pp = plot([ds.c df1.c dbf2.c dbf3.c ]);
          set(pp(1),'linewidth',4,'color','r');
          set(pp(2),'color','b','linewidth',1.5);
          set(pp(4),'color','k','linewidth',2,'linestyle','--');

        legend('TRUTH','nonlin filter','linear','nl-pred','location','best');
        title('credibility stock')
```

```
----------------------------------------------------------------
*** Starting the Exact NLS-Filter of the model. IN TESTING ***
----------------------------------------------------------------

                                          Norm of      First-order
 Iteration  Func-count     f(x)          step          optimality   CG-iterations
     0         215    1.60518e+020                      1.05e+020
     1         430    3.64957e+018        6.27094        3.2e+019            0
     2         645    3.64957e+018        6.17479        3.2e+019            0
     3         860    4.48584e+017         1.5437      6.18e+018            0
     4        1075    4.48584e+017         3.0874      6.18e+018            0
     5        1290    1.27257e+017       0.771849      4.48e+018            0
     6        1505    8.66464e+016         1.5437      4.19e+018            0
     7        1720    3.10208e+016         1.5437      3.04e+018            0
     8        1935    2.50651e+015       0.919648      8.46e+017            0
     9        2150    1.04295e+012      0.0839654       1.3e+016            0
    10        2365    4.17685e+010      0.0219612      1.94e+015            0
    11        2580         79293.4     0.00097352      6.02e+012            0
```

```
    12       2795          143.323   9.19271e-005      2.37e+010             0
    13       3010          140.927   2.68395e-007      5.68e+007             0

Solver stopped prematurely.

lsqnonlin stopped because it exceeded the function evaluation limit,
options.MaxFunEvals = 3000 (the selected value).

----------------------------------------------------------------
*** Filter CPU time: 1019.764631 seconds.
----------------------------------------------------------------
```



8 Conclusions

As can be seen, the exact non-linar filter achieves more precision in this particular case. In principle it should be this way in each case. However, the price one pays for the increased precision, often just by a little, is not small – ten minutes in this particular case on my super slow laptop. The sample of the simulation is realistic, but the size of the model and number of shocks is more on the smaller side than would be the case of models operated by central banks. In this case one would have to parallelize, which would speed up thing enormously as the problem is embarassignly parallel, or just run it over night....

Kind regards MA