

# System Priors for Econometric Time Series: An AR(2) Example

Michal Andrle, Miroslav Plašil

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## I. INTRODUCTION

The purpose of this short note is to give a simple and intuitive example of using non-mechanical **system priors** for time series models. The particular case will be for a AR(2) process, with two roots and one parameter for standard deviations of innovation.

System priors were introduced in [Andrle and Benes \(2013\)](#) as an alternative or supplement to commonly used independent marginal priors for structural economic models. System priors well-defined structural and economically meaningful priors about the model properties. The authors discuss an example of the sacrifice ratio prior for a Dynamic New-Keynesian model, which elicits joint prior distribution over a multiple of individual structural coefficients of the model. Importantly, system priors allow priors to be economically meaningful and easily communicated to a broader audience.

This note uses a much simpler and more educative example—a zero-mean autoregressive process of order two, AR(2). For details of the implementation using Random Walk Metropolis algorithm for more complex models, see [Andrle and Benes \(2013\)](#).

## II. SYSTEM PRIORS FOR AR(2)

We will focus only on two deterministic coefficients  $\phi_1$  and  $\phi_2$  of the AR(2) process

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2). \quad (1)$$

This is not really a useful process for macroeconomic time series.

What would be a reasonable prior for the two coefficients? Let's start with individual marginal priors with Normal distribution, i.e.  $\phi_1 \sim N(0, \sigma_{\phi_1}^2)$  and  $\phi_2 \sim N(0, \sigma_{\phi_2}^2)$ .

Does that sound about right? No way. The coefficients can move independently and the joint distribution is very wide, covering an array of dynamics, including wild oscillation, unstable non-stationary impulse-response functions, etc. Can we do better? You bet!

First, it is easy to see that the AR(2) process is stable only if  $\phi_1 + \phi_2 < 1$ ,  $\phi_2 - \phi_1 < 1$  and  $|\phi_2| < 1$ . This, naturally restricts the parameter space. How about oscillations and the degree of dampening? Just take a look at the roots of the associated characteristic equation and limit yourself to those combination of  $\phi_1$  and  $\phi_2$  that satisfy the relevant conditions. The implied prior distribution is easily implemented and implies nontrivial joint distribution of  $\phi_1, \phi_2$ . Fig. 1 shows the cross plot of the two parameters implied for stability, stability and real roots, and the spectral prior.

System priors pose very few restriction on the design of the prior—it is only the meaningfulness of the prior for the analyst and her audience that matters. Let's turn to another example, restriction on spectral characteristics of the process.

Let's elicit the joint prior distribution of  $\phi_1$  and  $\phi_2$  from such a distribution that updates independent Normal with the condition that at least 60% of variance of  $y_t$  is contributed from business cycle frequencies—6 to 32 periods. For this we need to know the spectral density of the process, since the integral of the spectral density is equal to unconditional variance of the process. Spectral density can be thus interpreted as a distribution of variance across frequencies.

The spectral density of  $y_t$ ,  $S_y(\omega)$ , is as follows:

$$S_y(\omega) = \frac{\sigma_\varepsilon^2}{2\pi[1 + \phi_1^2 + \phi_2^2 + 2\phi_1(\phi_2 - 1)\cos\omega - 2\phi_2\cos(2\omega)]}, \quad (2)$$

where  $\omega \in [0, \pi)$  is the angular frequency. The spectrum of the AR(2) process can take a wide number of shapes. For positive  $\phi_1$  and  $\phi_2$  it is easy to see that the peak is always at  $\omega = 0$ .

To implement our system prior, we define the total variance as the integral of the spectrum over full frequency range and business cycle variance as the integral only at the business cycle frequency range. Taking the ratio creates a statistics that is univariate, with clear units and relative interpretation. Further, a change in the standard deviation of  $\varepsilon$  only shifts the spectrum up or down but does not affect the ratio of cyclical variance to the total. As such, this system prior is completely uninformative about  $\sigma_\varepsilon^2$ .

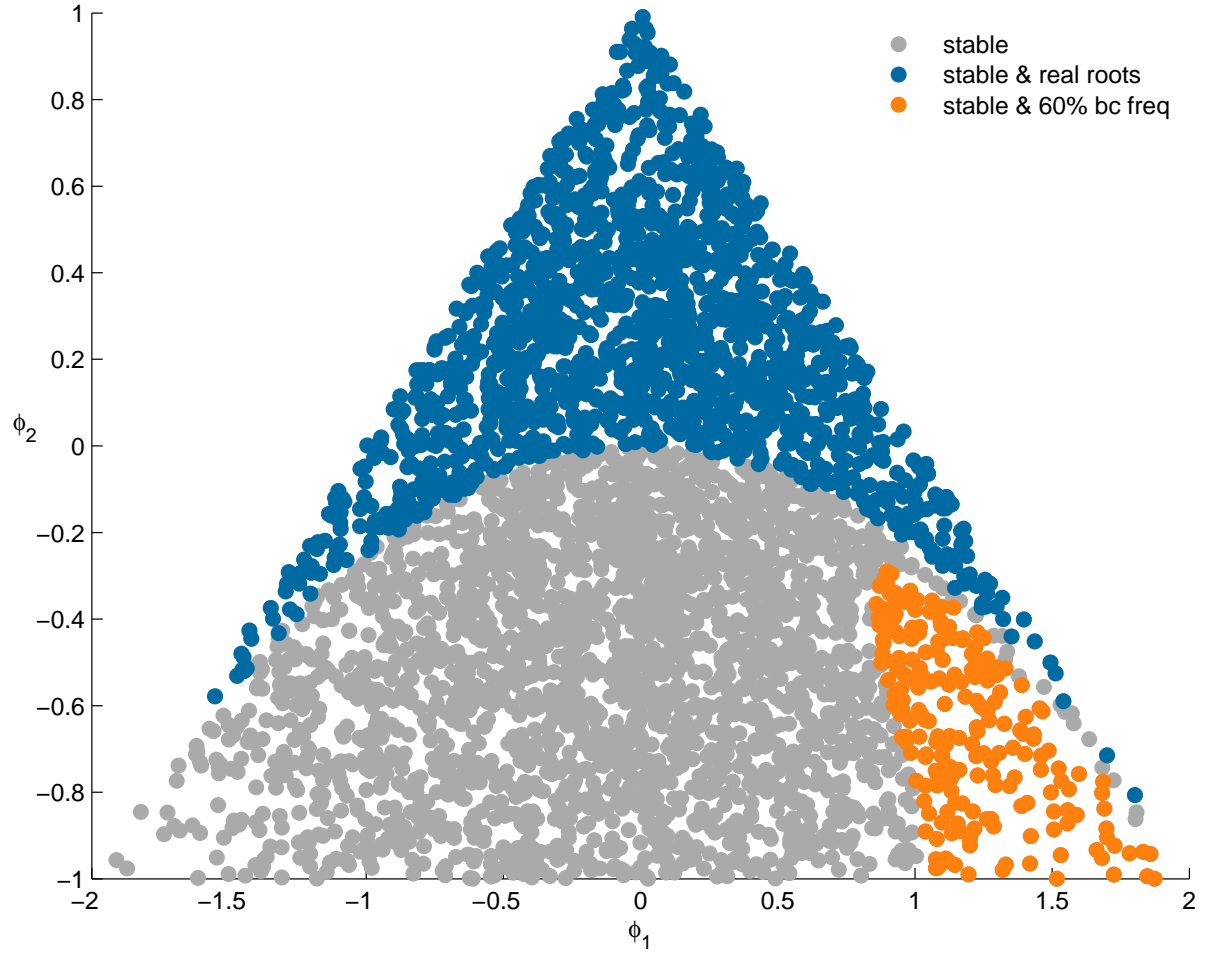
Fig. 1 demonstrates the set of parameters that conform to the stability system prior and also combinations of parameters that conform to the requirement of sufficient variance coming from business cycle frequencies.<sup>1</sup>

Knowing just the combinations—and the full joint distribution—of parameters that satisfy the constraint is not enough. The key knowledge is to understand how these translate to the behav-

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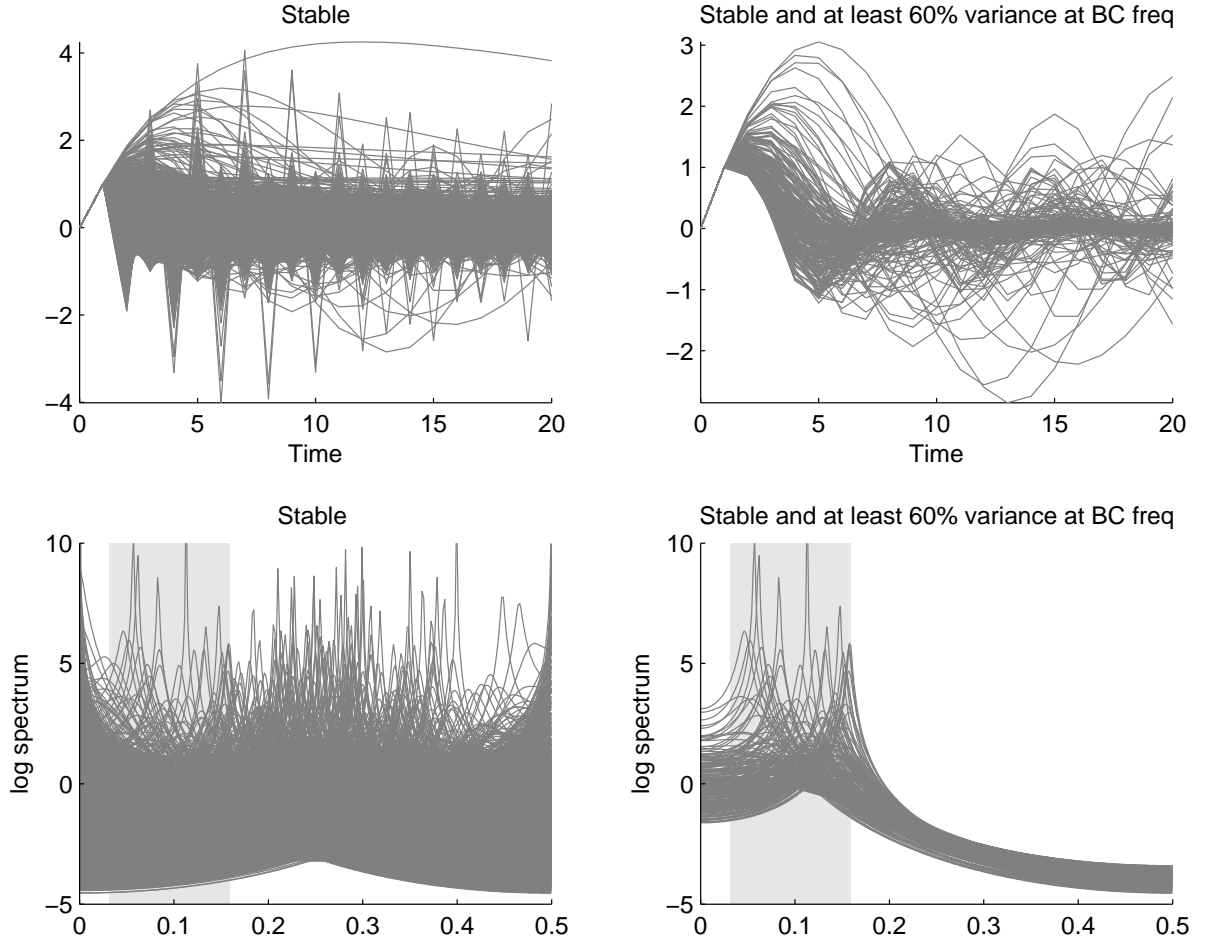
<sup>1</sup>For general formulation and estimation with system priors see [Andrle and Benes \(2013\)](#). In this case we used rejection sampling with bivariate Normal distribution as the proposal distribution.

**Figure 1. Admissible regions for the parameters**



ior of the model. For this purpose the prior-predictive distribution of models' properties need to be analyzed. For the AR(2) looking at the impulse-response function and the spectral density fits the bill.

Fig. 2 depicts impulse response functions and spectral densities for two system priors, i.e. for two admissible regions (with frequency driven by the distribution of parameters in the admissible region). One can easily see that the condition of stationarity does not restricts the process in an overly narrow way, while the spectral system prior limits the behavior of the model quite significantly. This system prior is not diffuse, it is very informative. It is also very transparent and easy to communicate and easy to agree or disagree with for the public.

**Figure 2. Model Properties for Admissible Regions**

Note: The business cycle frequencies denoted by the shaded region.

### III. CONCLUSION

This short note provided an example of using system priors for analysis and estimation of time series. The lessons from the Mickey Mouse AR(2) model are applicable and easily implementable for a large class of univariate or multivariate time series models, including Bayesian Vector AutoRegressions (BVARs) and structural BVARs.

Details of the implementation of system priors, further discussion, and an application with a Dynamic New-Keynesian model is provided in [Andrle and Benes \(2013\)](#) and the reader is invited to pursue the topic in detail.

**REFERENCES**

Andrle, Michal, and Jaromir Benes, 2013, “System Priors: Formulating Priors about DSGE Models System Properties,” Working Paper WP/13/257, International Monetary Fund, Washington DC.